



# Optimal versus observed physical capital across the world

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# Optimal capital accumulation in the Solow growth model

- Consider an economy that combines capital,  $K$ , and labor,  $L$ , to produce goods and services,  $Y$ , according to the production function:

$$Y = K^\alpha L^{1-\alpha}$$

- Capital accumulates with investment,  $I$ , and depreciates with the rate  $\delta$ , according to:

$$K_{+1} = (1 - \delta)K + I$$

- Consumption,  $C$ , is given between the difference of production and investment, according to:

$$C = Y - I$$

- Define variables in per capita terms with lower case variables such that  $y = \frac{Y}{L}$ ,  $k = \frac{K}{L}$ ,  $i = \frac{I}{L}$ ,  $c = \frac{C}{L}$

- Population grows according to:  $L_{+1} = (1 + n)L$

- Optimal capital,  $k^*$ , is given by the level that maximizes steady-state consumption,  $c = k^\alpha - (\delta + n)k$ , where the marginal productivity of capital,  $\alpha k^{\alpha-1}$ , equals its replacement cost,  $\delta + n$ , or:

$$k^* = \left[ \frac{\alpha}{\delta + n} \right]^{\frac{1}{1-\alpha}}$$

- From this expression it follows that optimal output,  $y^*$ , is given by:

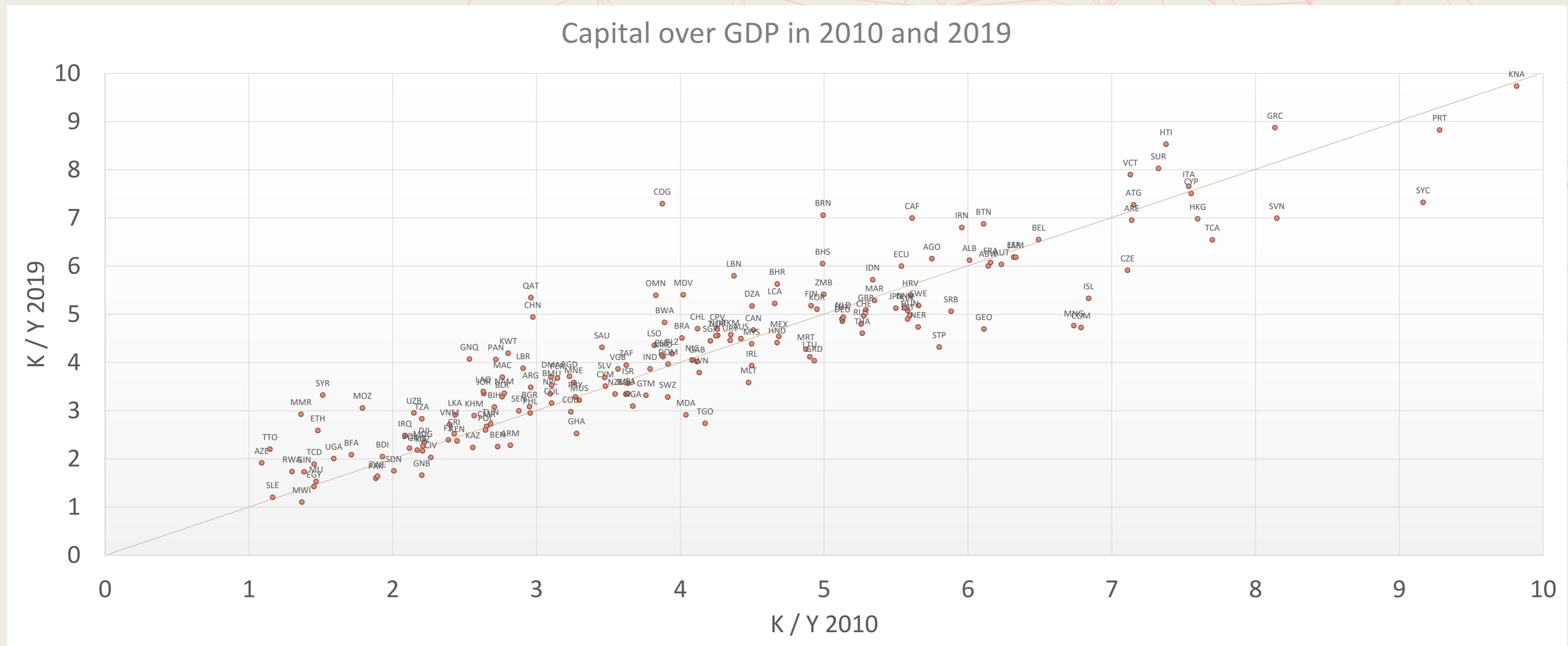
$$y^* = \left[ \frac{\alpha}{\delta + n} \right]^{\frac{\alpha}{1-\alpha}}$$

- and the optimal capital to output ratio is given by:

$$\frac{k^*}{y^*} = \frac{\alpha}{\delta + n}$$

Despite the fact that  $K/Y$  grew in most countries in the 2010 – 2019 period ...

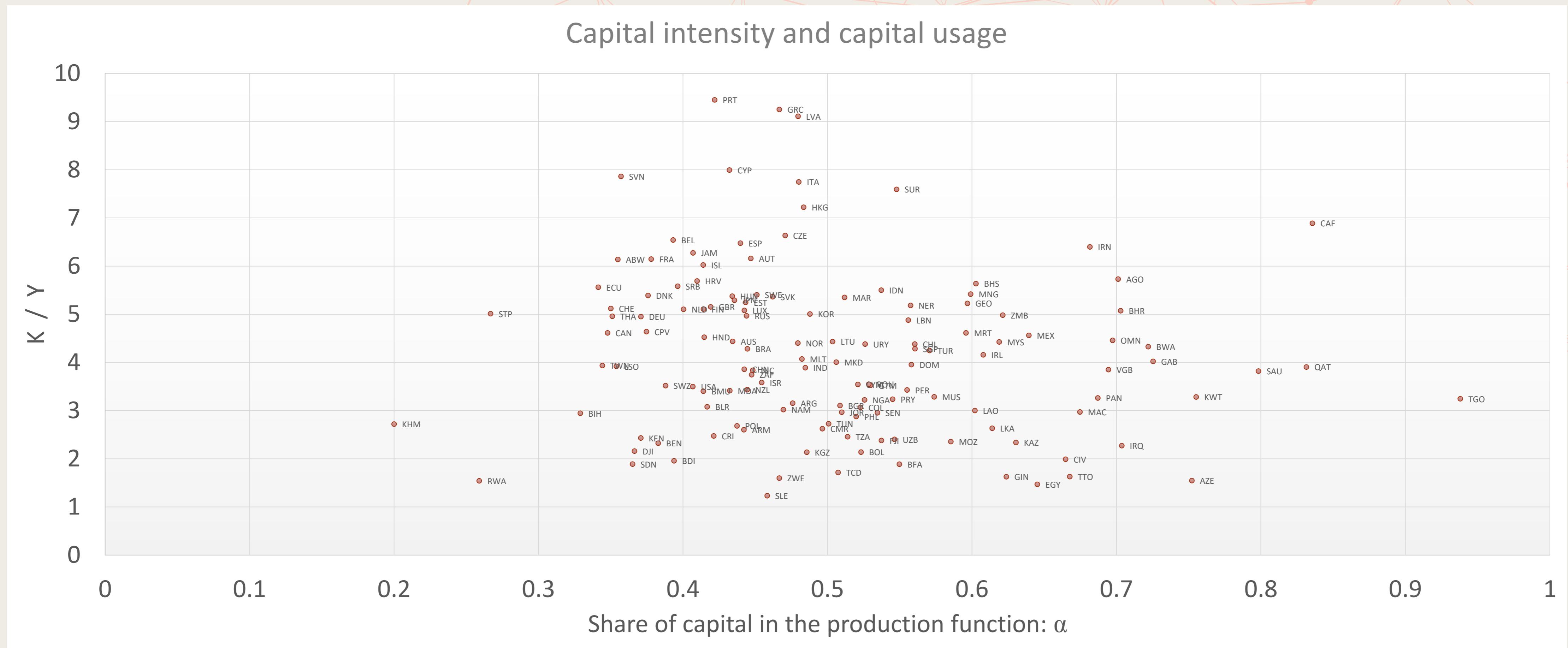
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**Source:** own calculations using data from Penn World Table version 10.0.

Feenstra, Robert C., Robert Inklaar and Marcel P. Timmer (2015), "The Next Generation of the Penn World Table" American Economic Review, 105(10), 3150-3182, available for download at <http://www.ggdc.net/pwt>.

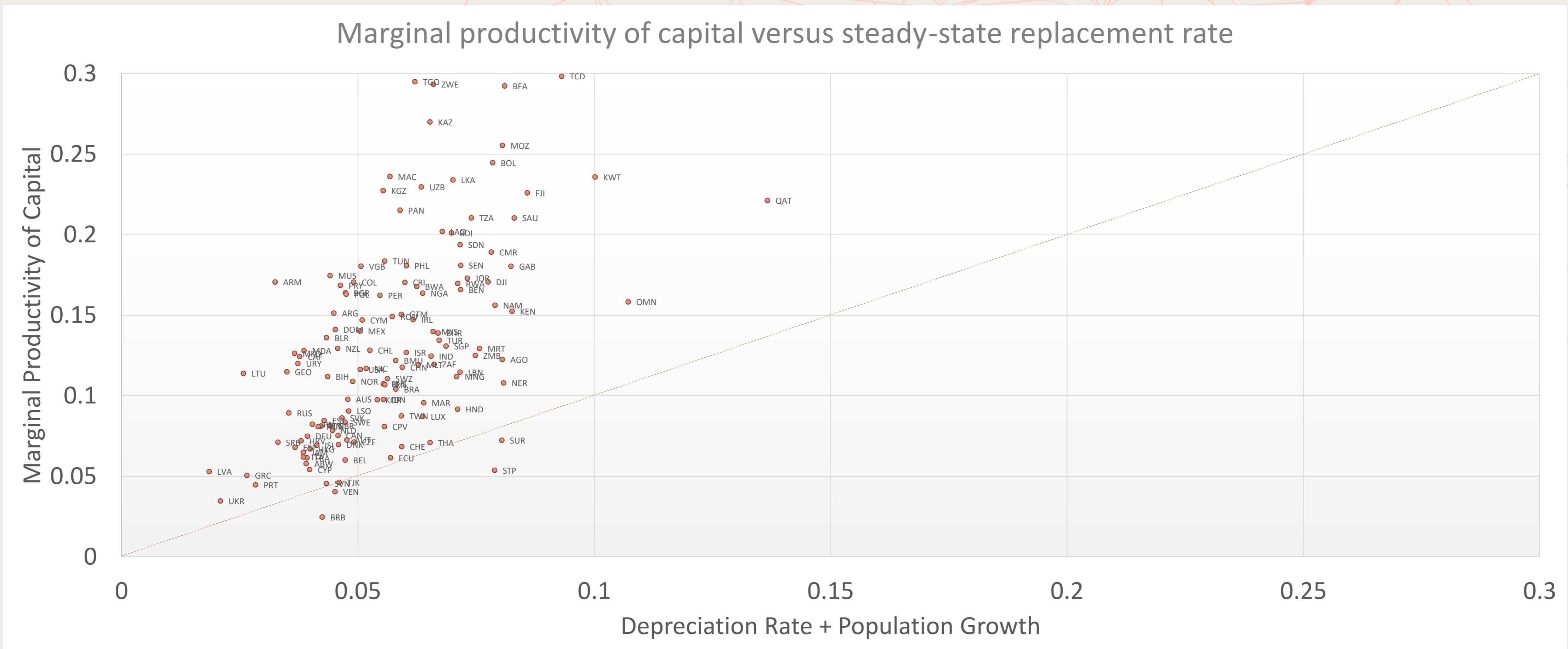
... given the capital intensity,  $\alpha$ , and the capital usage  $K/Y$ ...



Source: own calculations using data from Penn World Table version 10.0.

... marginal productivity of capital,  $\alpha Y/K$ , is still high, relative to the optimal,  $\delta + n$ , criteria

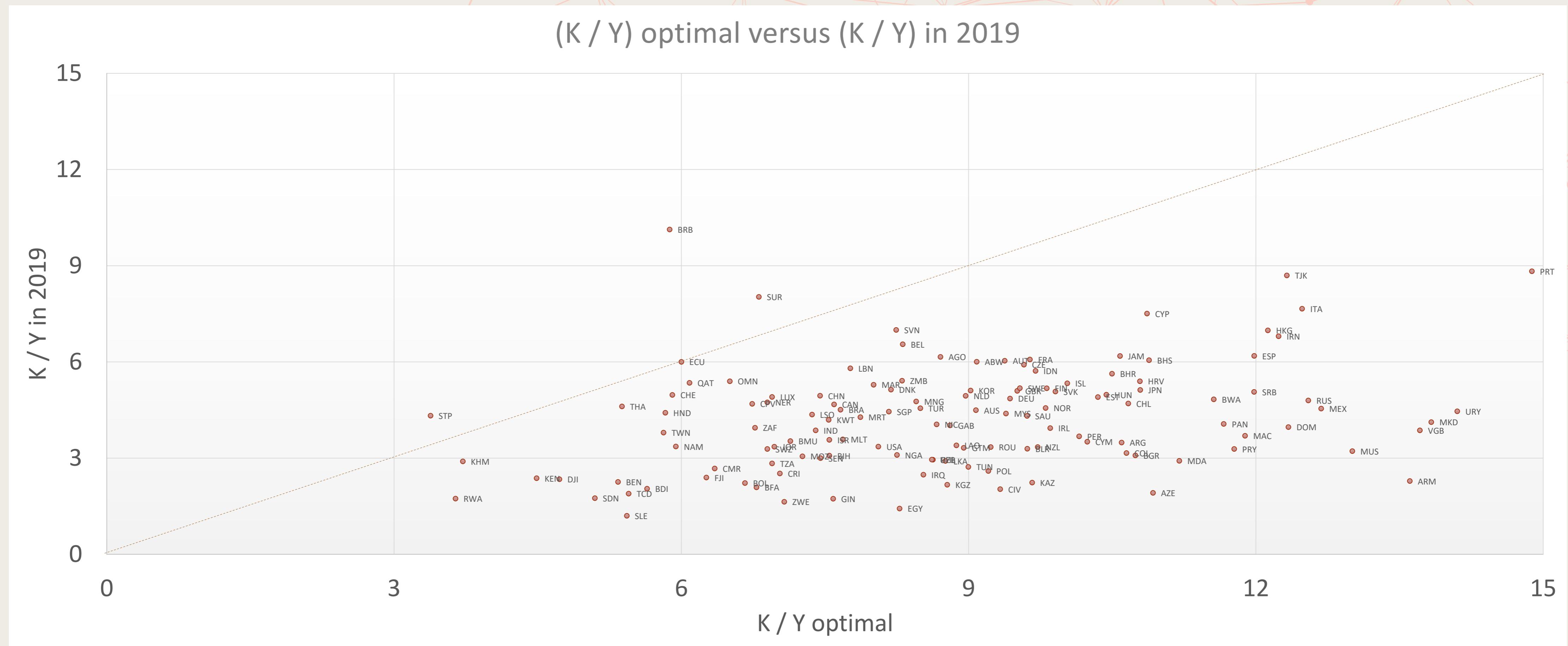
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**Source:** own calculations using data from Penn World Table version 10.0.

Therefore, the observed capital to output ratio is still below its optimal level

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**Source:** own calculations using data from Penn World Table version 10.0