



Yield Curves

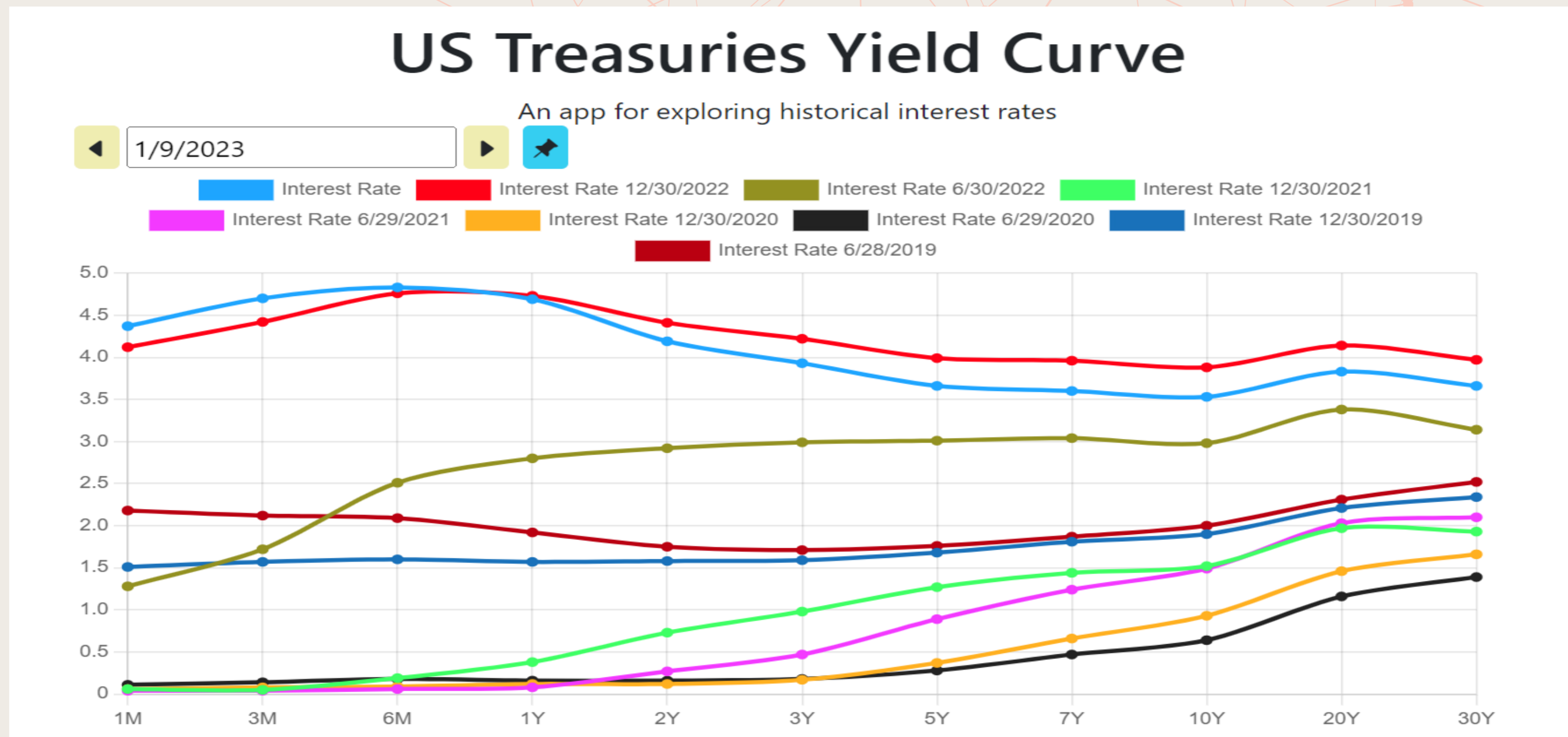
Alberto Ortiz Bolaños
January 18, 2023.

Partly based on Frederic Mishkin and Stanley Eakins “Financial Markets and Institutions”

Yield Curve



- The Yield Curve provides information about prevailing interest rates for debt securities of different tenors at a particular point in time.



Yield Curves Around the World



Country	1 year	2 year	5 year	10 year
Australia	3.11%	3.06%	3.18%	3.42%
Austria	2.64%	2.53%	2.30%	2.49%
Bahrain	6.13%	5.81%	5.68%	5.46%
Bangladesh	8.12%	8.38%	8.47%	8.51%
Belgium	2.73%	2.34%	2.25%	2.48%
Brazil	13.54%	12.54%	12.38%	12.52%
Bulgaria	2.28%	2.78%	3.79%	5.48%
Canada	4.45%	3.51%	2.83%	2.76%
Chile	8.98%	7.85%	5.30%	5.29%
China	2.11%	2.36%	2.70%	2.96%
Colombia	9.27%	10.18%	12.10%	12.33%
Croatia	2.81%	2.97%	3.33%	3.65%
Cyprus	3.03%	2.91%	3.20%	4.03%
Czech Republic	6.62%	5.57%	4.62%	4.18%
Denmark	2.33%	2.61%	2.25%	2.22%
Egypt	20.13%	20.01%	19.64%	19.21%
Finland	2.58%	2.41%	2.36%	2.46%
France	2.71%	2.44%	2.27%	2.40%
Germany	2.57%	2.43%	2.01%	1.97%
Greece	3.15%	3.27%	3.13%	3.98%
Hong Kong	3.64%	3.65%	3.11%	3.08%
Hungary	13.61%	11.68%	8.03%	7.33%
Iceland	7.10%	7.09%	7.06%	6.45%
India	6.74%	6.94%	7.16%	7.33%
Indonesia	5.72%	6.02%	6.51%	6.81%
Ireland	2.46%	2.37%	2.23%	2.36%
Israel	3.93%	3.49%	3.19%	3.13%
Italy	2.87%	2.77%	3.18%	3.70%
Japan	-0.02%	0.01%	0.24%	0.42%
Kazakhstan	16.51%	16.50%	15.45%	13.94%
Kenya	10.37%	11.68%	13.95%	14.35%
Latvia	2.10%	3.00%	4.10%	4.60%
Lithuania	3.90%	3.95%	4.10%	4.65%
Malaysia	3.18%	3.38%	3.76%	3.97%
Malta	4.15%	3.74%	3.36%	2.82%

Country	1 year	2 year	5 year	10 year
Mauritius	4.10%	4.38%	5.07%	5.82%
Mexico	11.04%	10.56%	9.10%	8.95%
Morocco	3.38%	3.77%	4.00%	4.20%
Namibia	9.03%	8.02%	6.10%	11.33%
Netherlands	2.25%	2.29%	2.15%	2.25%
New Zealand	5.00%	4.54%	4.11%	4.12%
Nigeria	5.43%	10.66%	13.90%	13.51%
Norway	3.04%	2.99%	2.70%	2.75%
Pakistan	15.80%	16.41%	15.65%	14.48%
Perù	6.85%	6.90%	7.02%	8.09%
Philippines	5.13%	5.55%	6.10%	6.33%
Poland	5.73%	5.73%	5.72%	5.80%
Portugal	2.51%	2.52%	2.39%	2.84%
Qatar	5.82%	5.36%	4.56%	4.09%
Romania	6.85%	6.73%	7.28%	7.45%
Russia	12.85%	8.94%	9.65%	10.45%
Serbia	5.02%	5.36%	6.30%	7.00%
Singapore	4.20%	3.15%	2.83%	2.89%
Slovakia	2.75%	2.76%	2.79%	2.81%
Slovenia	1.54%	1.75%	2.55%	2.98%
South Africa	7.61%	7.55%	8.14%	9.66%
South Korea	3.56%	3.46%	3.36%	3.35%
Spain	2.79%	2.63%	2.55%	2.92%
Sri Lanka	28.28%	32.73%	28.99%	28.56%
Sweden	2.73%	2.54%	2.00%	1.92%
Switzerland	1.31%	0.94%	0.89%	0.98%
Taiwan	0.95%	0.98%	1.06%	1.23%
Thailand	1.71%	1.78%	1.90%	2.37%
Turkey	13.74%	10.30%	9.40%	10.08%
Uganda	10.02%	11.56%	14.70%	15.68%
Ukraine	64.07%	23.17%	25.77%	17.71%
United Kingdom	3.61%	3.45%	3.26%	3.29%
United States	4.66%	4.10%	3.46%	3.39%
Vietnam	4.54%	4.54%	4.62%	4.75%

Source: <http://www.worldgovernmentbonds.com/inverted-yield-curves/>

Term Structure of Interest Rates: Facts to be Explained



1. Interest rates for different maturities move together.
2. Yield curves tend to have steep upward slope when short rates are low and downward slope when short rates are high.
3. Yield curve is typically upward sloping.

Three Theories of Term Structure



A. Pure Expectations Theory

- Pure Expectations Theory explains 1 and 2, but not 3.

B. Market Segmentation Theory

- Market Segmentation Theory explains 3, but not 1 and 2

C. Liquidity Premium Theory

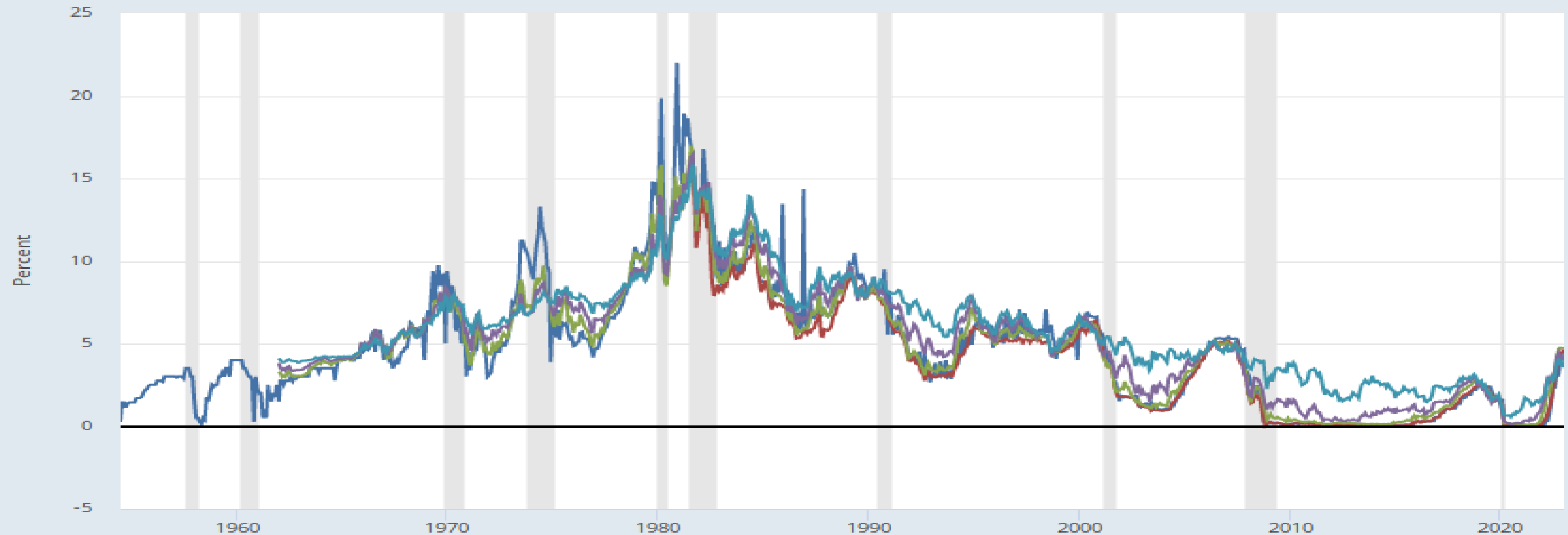
- Solution: Combine features of both Pure Expectations Theory and Market Segmentation Theory to get Liquidity Premium Theory and explain all facts.

Interest Rates on Different Maturity Bonds Move Together



FRED 

- Federal Funds Effective Rate
- Market Yield on U.S. Treasury Securities at 3-Month Constant Maturity, Quoted on an Investment Basis
- Market Yield on U.S. Treasury Securities at 1-Year Constant Maturity, Quoted on an Investment Basis
- Market Yield on U.S. Treasury Securities at 3-Year Constant Maturity, Quoted on an Investment Basis
- Market Yield on U.S. Treasury Securities at 10-Year Constant Maturity, Quoted on an Investment Basis



Source: Board of Governors of the Federal Reserve System (US)

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A. Pure Expectations Theory



- **Key Assumption:** Bonds of different maturities are perfect substitutes
- **Implication:** expected return, R^e , on bonds of different maturities are equal (arbitrage)
- Investment strategies for two-period horizon
 1. Buy \$1 of one-year bond and when matures buy another one-year bond.
 2. Buy \$1 of two-year bond and hold it.

No-Arbitrage condition



Expected Return from Investment Strategy 1: two one-year bonds

$$\begin{aligned} & (1 + i_{1,t})(1 + i_{1,t+1}^e) - 1 \\ & = i_{1,t} + i_{1,t+1}^e + i_{1,t}i_{1,t+1}^e \end{aligned}$$

Expected Return from Investment Strategy 2: one two-years bond

$$\begin{aligned} & (1 + i_{2,t})(1 + i_{2,t}) - 1 \\ & = 2i_{2,t} + i_{2,t}^2 \end{aligned}$$

No-Arbitrage condition:: $i_{1,t} + i_{1,t+1}^e + i_{1,t}i_{1,t+1}^e = 2i_{2,t} + i_{2,t}^2$

Under the assumption that interest rates are small, the product of two small numbers is negligible, therefore $i_{1,t}i_{1,t+1}^e \approx 0$ and $i_{2,t}^2 \approx 0$, which simplifies the arbitrage condition to: $i_{1,t} + i_{1,t+1}^e \approx 2i_{2,t}$

Interest Rates for Long Tenors



- Under this assumption, solving for $i_{2,t}$ we have:

$$i_{2,t} = \frac{i_{1,t} + i_{1,t+1}^e}{2}$$

- Therefore, the interest rate for a two-year bond equals the average of the observed interest rate of a one-year bond prevailing today and the expected interest rate of a one-year bond next year.
- More generally, for an n -period bond, the interest rate is determined by the average short-term rates expected to occur during the life of the long bond

$$i_{n,t} = \frac{i_{1,t} + i_{1,t+1}^e + i_{1,t+2}^e + \cdots + i_{1,t+n-1}^e}{n}$$



Interest Rates for Long Tenors: Numerical Example

- Numerical example
 - One-year interest rate over the next five years is expected to be 5%, 6%, 7%, 8%, and 9%
- Interest rate on two-year bond:
 $(5\% + 6\%)/2 = 5.5\%$
- Interest rate for five-year bond:
 $(5\% + 6\% + 7\% + 8\% + 9\%)/5 = 7\%$
- Interest rate for one- to five-year bonds:
5%, 5.5%, 6%, 6.5% and 7%

Pure Expectations Theory and Term Structure Facts

- Explains why yield curve has different slopes
 - i. When short rates are expected to rise in future, average of future short rates = i_{nt} is above today's short rate; therefore yield curve is upward sloping.
 - ii. When short rates expected to stay same in future, average of future short rates same as today's, and yield curve is flat.
 - iii. Only when short rates expected to fall will yield curve be downward sloping.

- 1. Interest rates for different maturities move together. ✓
 - i. Short rate rises are persistent
 - ii. If $i_t \uparrow$ today, i_{t+1}^e, i_{t+2}^e etc. $\uparrow \Rightarrow$ average of future rates $\uparrow \Rightarrow i_{nt} \uparrow$
 - iii. Therefore: $i_t \uparrow \Rightarrow i_{nt} \uparrow$ (i.e., short and long rates move together)

- 2. Yield curves tend to have steep upward slope when short rates are low and downward slope when short rates are high. ✓
 - i. When short rates are low, they are expected to rise to normal level, and long rate = average of future short rates will be well above today's short rate; yield curve will have steep upward slope.
 - ii. When short rates are high, they will be expected to fall in future, and long rate will be below current short rate; yield curve will have downward slope.

- 3. Yield curve is typically upward sloping. ✗
 - i. Short rates as likely to fall in the future as rise, therefore nothing guarantees an upward sloping yield curve.

B. Market Segmentation Theory



- **Key Assumption:** Bonds of different maturities are NOT substitutes at all.
- **Implication:** Markets are completely segmented and interest rate at each maturity are determined separately
- Explains fact 3—that yield curve is usually upward sloping
 - People typically prefer short holding periods and thus have higher demand for short-term bonds, which have higher prices and lower interest rates than long bonds.
- Does not explain fact 1 or fact 2 because it assumes long-term and short-term rates are determined independently.

C. Liquidity Premium Theory

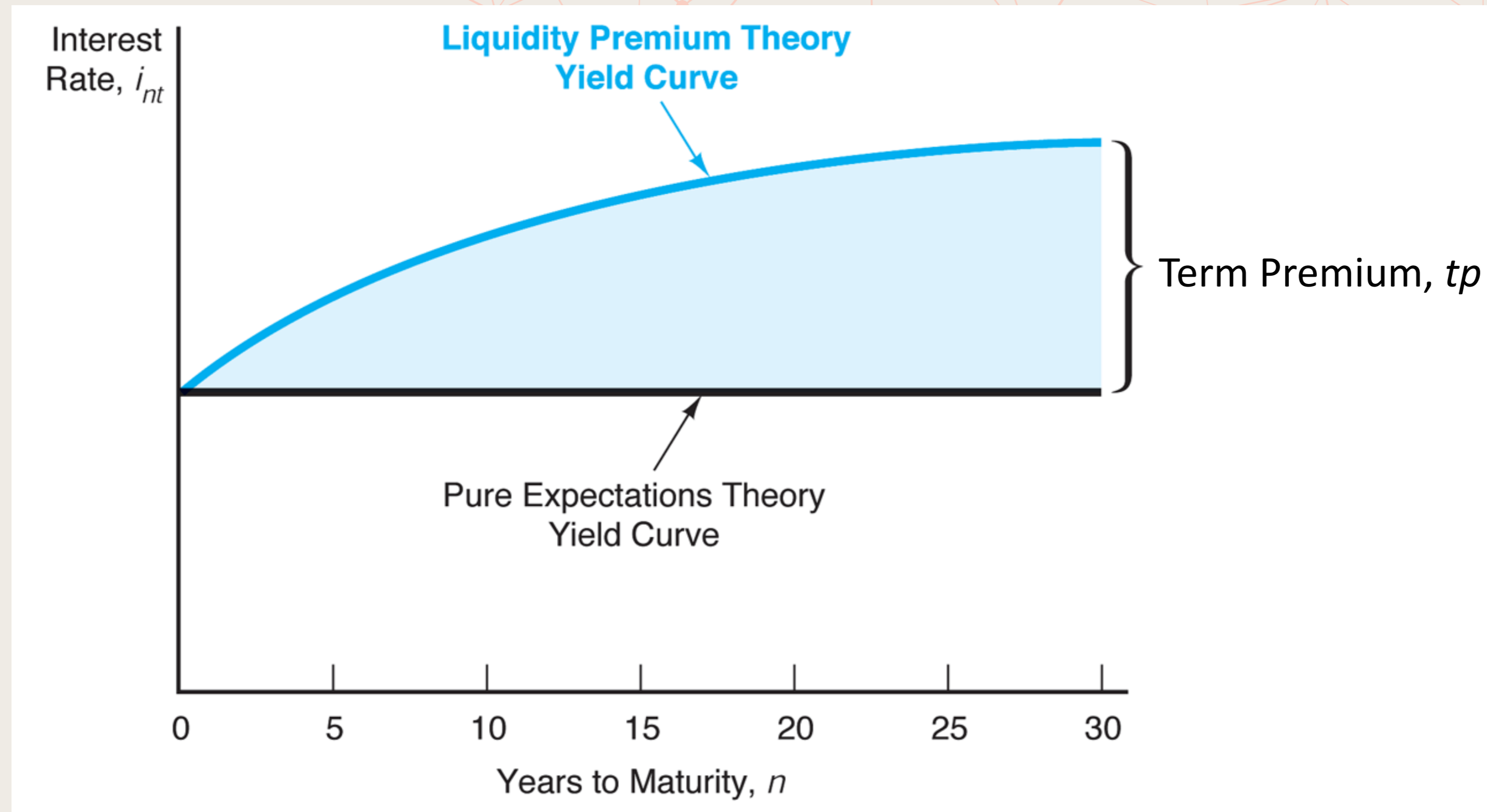


- **Key Assumption:** Bonds of different maturities are substitutes but are not perfect substitutes.
- **Implication:** Modifies Pure Expectations Theory with features of Market Segmentation Theory.
- Investors prefer short rather than long bonds \Rightarrow must be paid positive term premium, $tp_{n,t}$, to hold long term bonds.
- This term premium is the extra compensation required by investors for bearing interest rate risk associated with short-term yields not evolving as expected.
- Results in following modification of Pure Expectations Theory

$$i_{n,t} = \frac{i_{1,t} + i_{1,t+1}^e + i_{1,t+2}^e + \dots + i_{1,t+n-1}^e}{n} + tp_{n,t}$$

Relationship between the Pure Expectations Theory and the Liquidity Premium Theory

$$i_{n,t} = \frac{i_{1,t} + i_{1,t+1}^e + i_{1,t+2}^e + \dots + i_{1,t+n-1}^e}{n} + tp_{n,t}$$

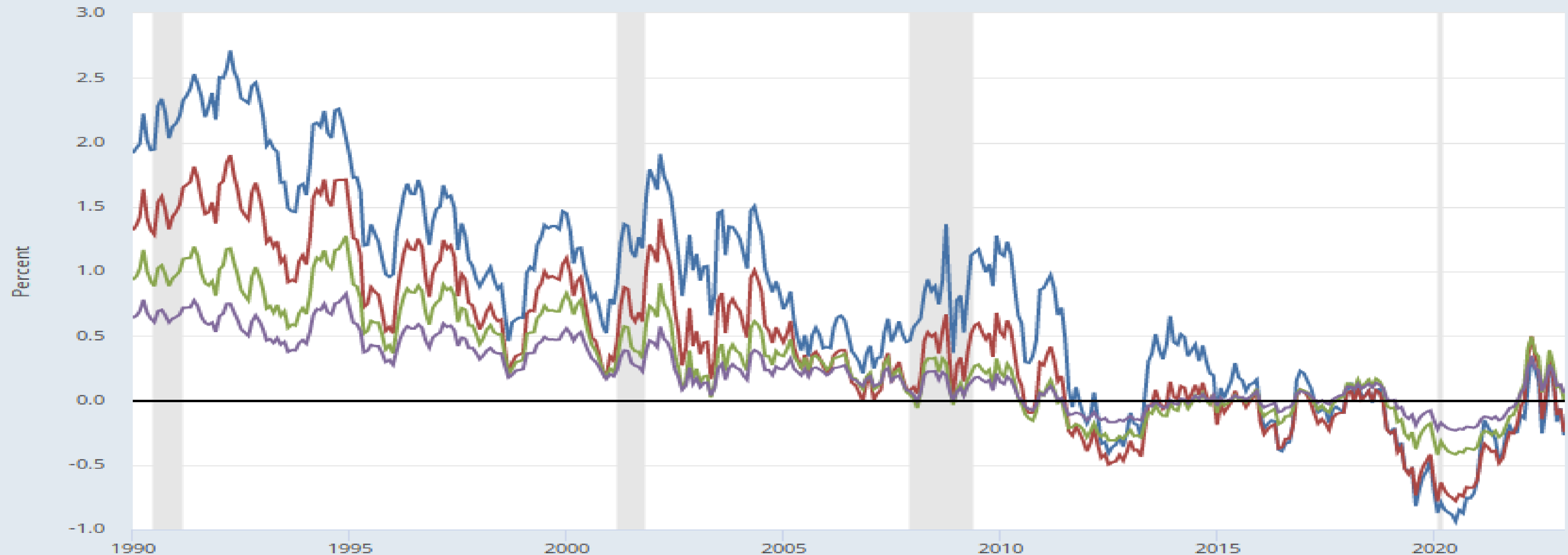


Term Premium on Different Maturity Bonds



FRED

- Term Premium on a 10 Year Zero Coupon Bond
- Term Premium on a 5 Year Zero Coupon Bond
- Term Premium on a 2 Year Zero Coupon Bond
- Term Premium on a 1 Year Zero Coupon Bond



Source: Board of Governors of the Federal Reserve System (US)

myf.red/g/YFRM

Data on the nominal term structure model from Kim and Wright

www.bankandfinance.net



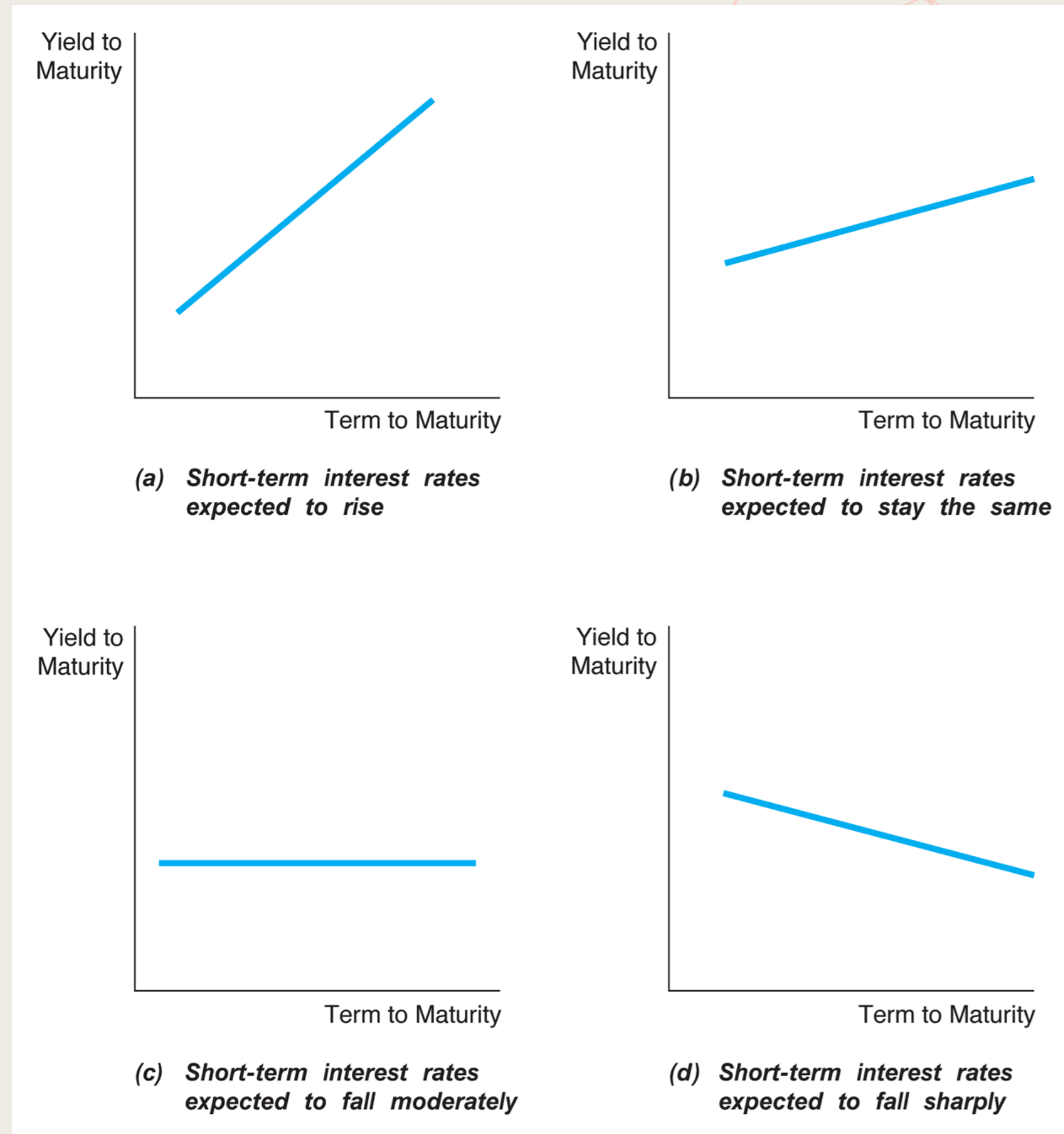
Interest Rates for Long Tenors: Numerical Example

- Numerical example
 - One-year interest rate over the next five years is expected to be 5%, 6%, 7%, 8%, and 9%
- Investors' preferences for holding short-term bonds so term premium for one- to five-year bonds: 0%, 0.25%, 0.5%, 0.75%, and 1.0%
- Interest rate on two-year bond:
 $(5\% + 6\%)/2 + 0.25\% = 5.75\%$
- Interest rate for five-year bond:
 $(5\% + 6\% + 7\% + 8\% + 9\%)/5 + 1.0\% = 8\%$
- Interest rate for one- to five-year bonds:
5%, 5.75%, 6.5%, 7.25% and 8%
- Comparing with those for the Pure Expectations Theory, Liquidity Premium Theory produces yield curves more steeply upward sloped.

Liquidity Premium Theory and Term Structure Facts

1. Interest rates for different maturities move together. ✓
 - i. Same reasons as the Pure Expectations Theory.
2. Yield curves tend to have steep upward slope when short rates are low and downward slope when short rates are high. ✓
 - i. Same reasons as the Pure Expectations Theory.
3. Yield curve is typically upward sloping. ✓
 - Upward sloped yield curve by liquidity premium for long-term bonds.

Yield Curves and the Market's Expectations of Future Short-Term Interest Rates



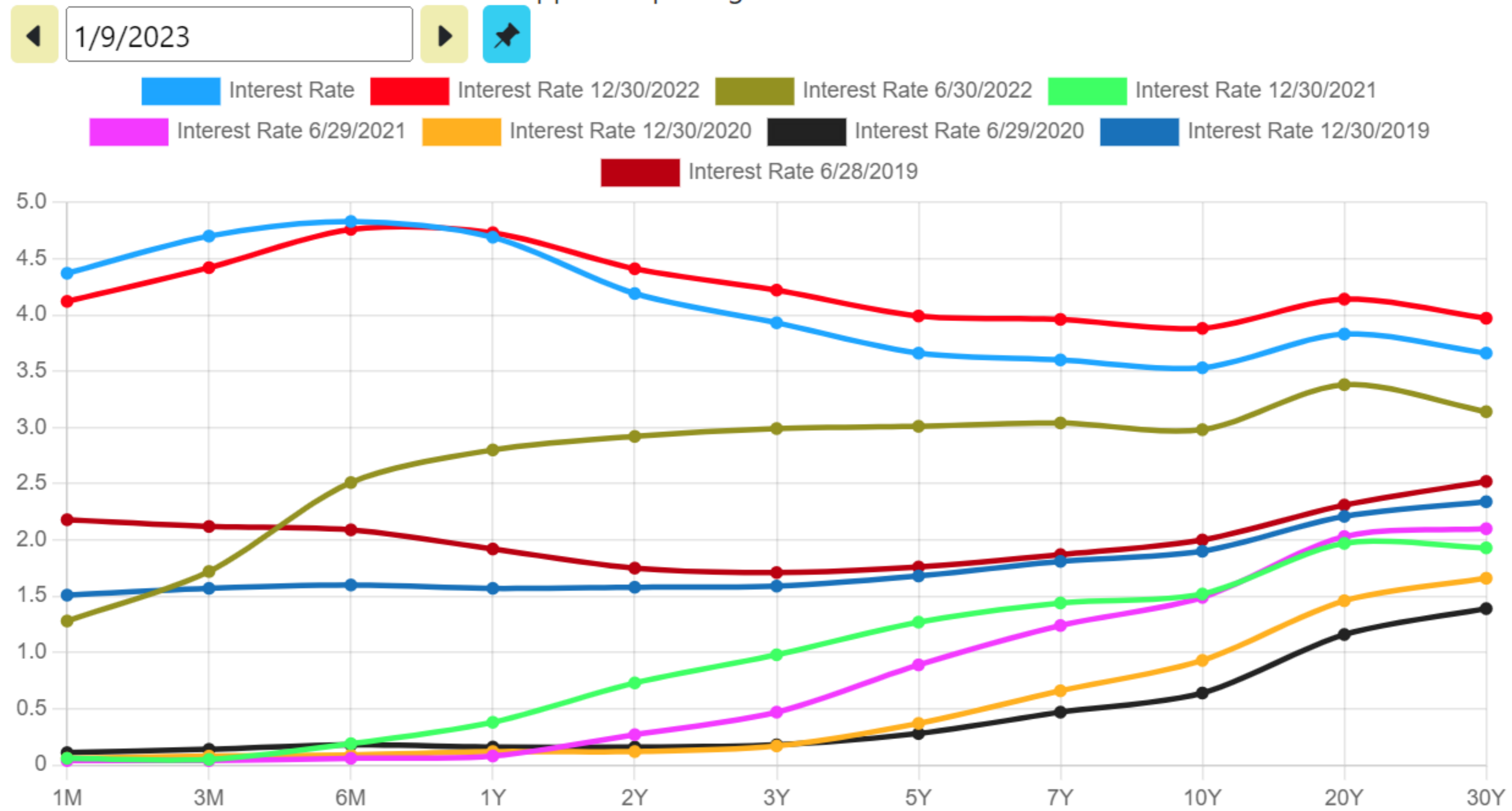
- An upward sloping yield curve:
 - i. is an interest rate environment in which long-term bonds have a higher yield than short-term ones,
 - ii. signals that short-term rates are expected to increase in the future,
 - iii. could signal forthcoming inflation.
- On the other hand, a downward sloping, or inverted yield curve:
 - i. is an interest rate environment in which long-term bonds have a lower yield than short-term ones,
 - ii. signals that short-term rates are expected to decrease in the future,
 - iii. is often considered a predictor of economic recession.
- As of 18 January 2023, 31 countries have an inverted yield curve.

Interpreting Yield Curves



US Treasuries Yield Curve

An app for exploring historical interest rates



Inverted Yield Curve



S&P Rating	Yield Curves			
	Totally Inverted	Partially Inverted	Minimally Inverted	
A to AAA	Chile			
	Czech Republic			
	Qatar			
	Malta			
	Canada			
	United States			
	Iceland			
	Sweden			
	Hong Kong	Switzerland		
	Germany	Finland		
	New Zealand	Poland	Australia	
	Denmark	Belgium		
	Israel	Spain		
	Singapore			
	Norway			
	United Kingdom			
	South Korea			
	Austria			
	Netherlands			
	France			
Ireland				
B to BBB	Hungary		Romania	
	Kazakhstan		Italy	
	Mexico	Portugal	Cyprus	
	Egypt	Greece	South Africa	
	Bahrain			
	Turkey			
	Brazil			
	Other	Ukraine	Namibia	Russia
		Sri Lanka		
		Pakistan		

Country	Long vs Short Term 10Y vs 2Y Spread	Mid vs Short Term 5Y vs 2Y Spread	Short Term 2Y vs 1Y Spread
Ukraine	-546.3 bp	260.3 bp	-4090.0 bp
Hungary	-434.5 bp	-364.5 bp	-193.5 bp
Sri Lanka	-417.6 bp	-374.2 bp	444.8 bp
Chile	-256.0 bp	-255.0 bp	-113.0 bp
Kazakhstan	-255.9 bp	-104.9 bp	-0.8 bp
Pakistan	-192.9 bp	-75.5 bp	60.9 bp
Mexico	-160.8 bp	-146.4 bp	-47.6 bp
Czech Republic	-139.7 bp	-95.9 bp	-104.8 bp
Qatar	-127.3 bp	-80.4 bp	-46.1 bp
Malta	-92.8 bp	-38.4 bp	-40.1 bp
Egypt	-79.9 bp	-37.2 bp	-12.5 bp
Canada	-75.1 bp	-67.4 bp	-94.3 bp
United States	-70.5 bp	-63.8 bp	-56.6 bp
Iceland	-64.1 bp	-3.0 bp	-1.0 bp
Sweden	-61.5 bp	-53.8 bp	-18.9 bp
Hong Kong	-57.0 bp	-53.7 bp	1.2 bp
Germany	-45.4 bp	-41.4 bp	-14.7 bp
New Zealand	-42.0 bp	-42.6 bp	-46.0 bp
Denmark	-39.1 bp	-36.0 bp	27.8 bp
Israel	-36.1 bp	-30.6 bp	-43.1 bp
Bahrain	-35.5 bp	-13.3 bp	-31.5 bp
Singapore	-25.7 bp	-31.7 bp	-104.6 bp
Norway	-24.1 bp	-29.0 bp	-5.0 bp
Turkey	-22.0 bp	-90.0 bp	-343.5 bp
United Kingdom	-15.6 bp	-19.0 bp	-16.1 bp
South Korea	-10.6 bp	-10.0 bp	-10.7 bp
Austria	-4.2 bp	-23.3 bp	-10.3 bp
Netherlands	-4.0 bp	-14.3 bp	4.1 bp
France	-3.8 bp	-16.5 bp	-27.5 bp
Brazil	-2.2 bp	-16.2 bp	-100.4 bp
Ireland	-1.1 bp	-13.6 bp	-9.1 bp

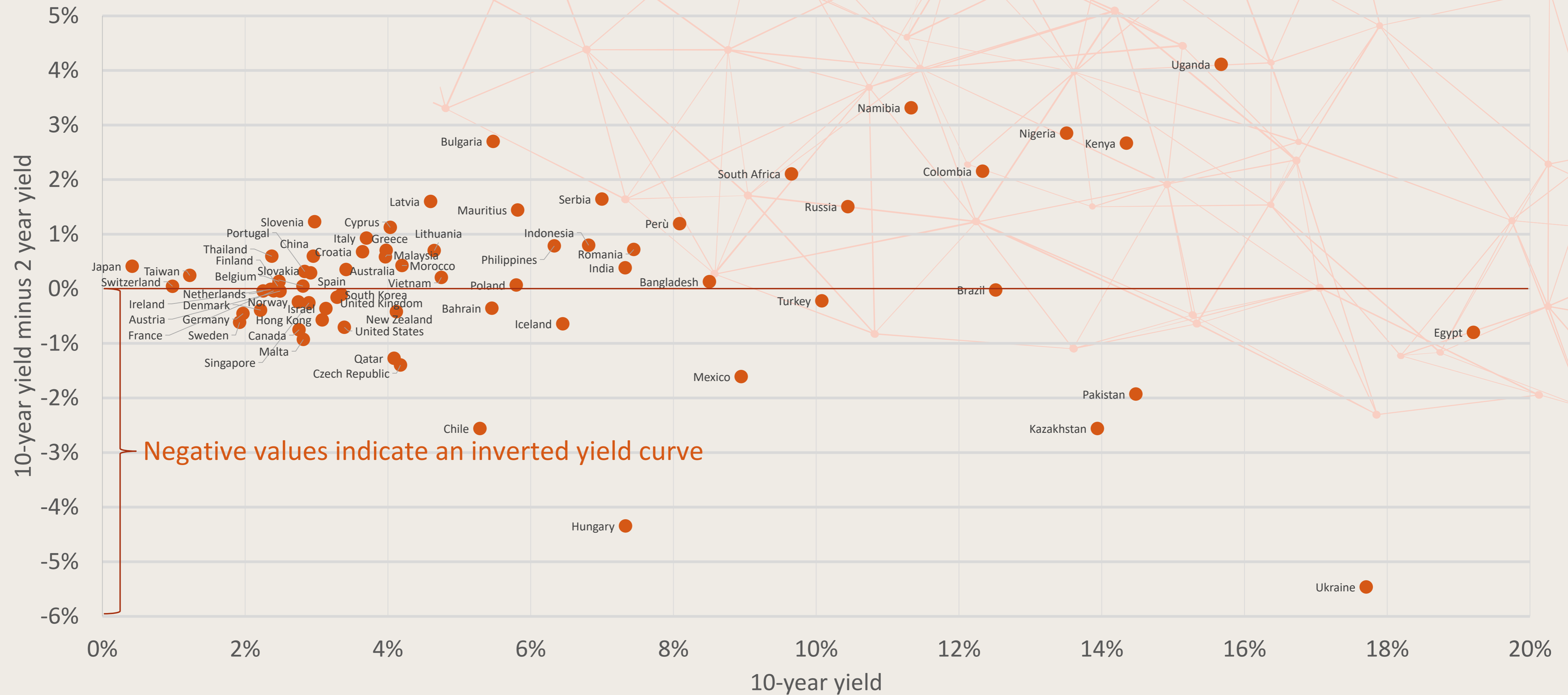
The convexity of the yield curve can be estimated calculating the spread between Government Bonds with long, medium and short maturity.

If the spread between the 10 years and the 2 years Government Bond is negative, it's a strong signal of totally inverted yield curve.

Signals of partially or minimally inverted yield curve are a negative 5Y vs 2Y spread or a negative 2Y vs 1Y spread.

Source:
<http://www.worldgovernmentbonds.com/inverted-yield-curves/>

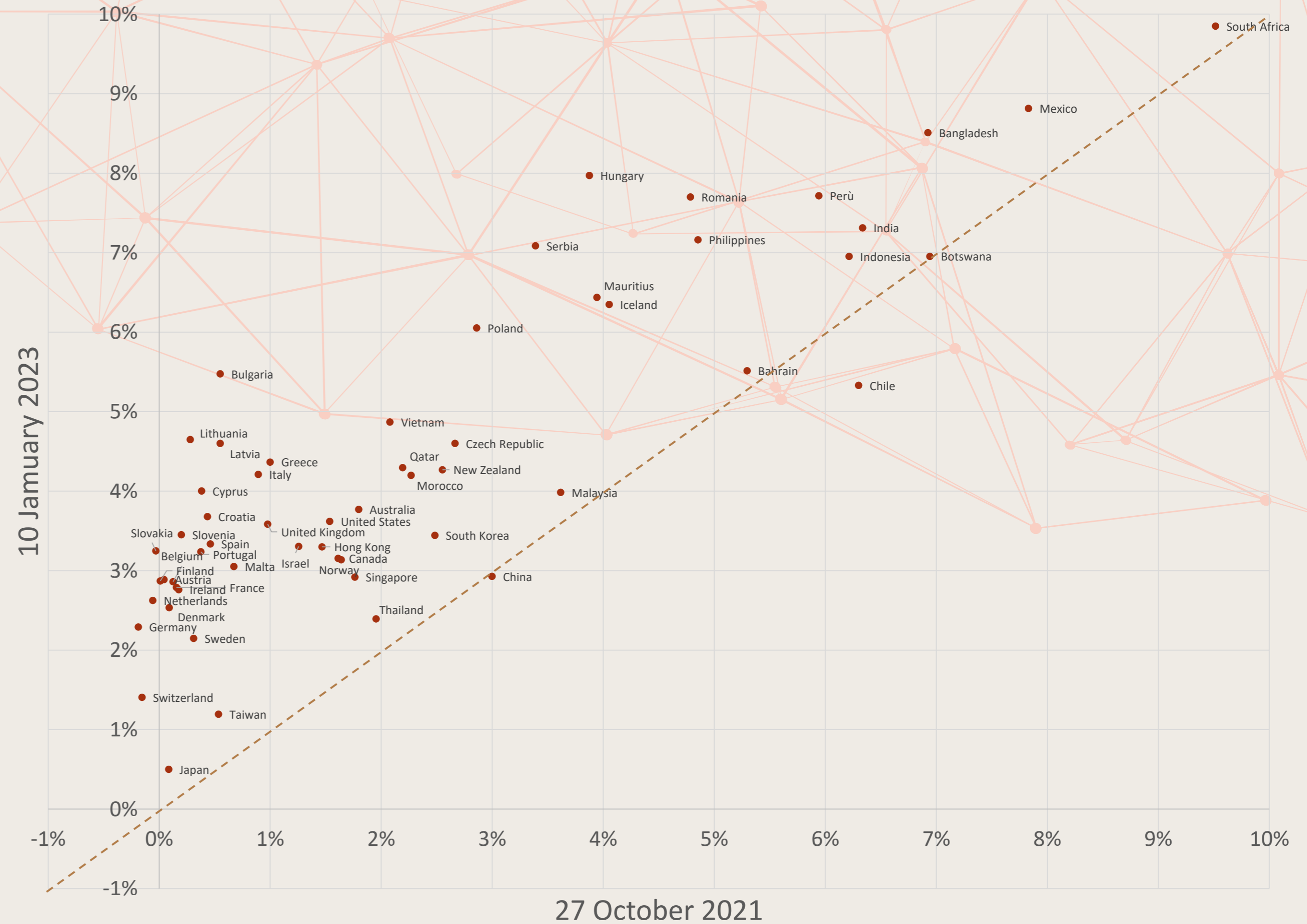
Government securities: 10-year yield and (10-year yield minus 2-year yield)



10 - Year Bond Yield



These Yield Curve inversions have taken place despite the fact that long-term rates have increased, as short-term rates have increased much more.



Forecasting Interest with Term Structure



- Which is today's market expectation for the one-year bond interest rates that will prevail next year, $i_{1,t+1}^e$?
- We know the current one- and two-years rates are $i_{1,t} = 5\%$ and $i_{2,t} = 5.5\%$, respectively, and the term premium on a two-year bond is $tp_{2,t} = 0.25\%$,

- The no-arbitrage condition, with the liquidity premium, requires:

$$(1 + i_{1,t})(1 + i_{1,t+1}^e) = (1 + i_{2,t} - tp_{2,t})(1 + i_{2,t} - tp_{2,t})$$

- Solving for $i_{1,t+1}^e$ we have:

$$i_{1,t+1}^e = \frac{(1 + i_{2,t} - tp_{2,t})(1 + i_{2,t} - tp_{2,t})}{(1 + i_{1,t})} - 1 = \frac{(1 + 5.5\% - 0.25\%)(1 + 5.5\% - 0.25\%)}{(1 + 5\%)} - 1 = 5.5\%$$

Forecasting Interest with Term Structure



- Which is today's market expectation for the one-year bond interest rates that will prevail in two years, $i_{1,t+2}^e$?
- We know the current one-, two- and three-years rates are $i_{1,t} = 5\%$, $i_{2,t} = 5.5\%$ and $i_{3,t} = 6\%$, respectively. We also know that expected one-year bond interest rate next year is $i_{1,t+1}^e = 5.5\%$ and that the term premium for three-year bonds is 0.5%.
- No-arbitrage requires that an investor gets the same expected return in three one-year bonds or in one three-years bond, which gives the condition:

$$(1 + i_{1,t})(1 + i_{1,t+1}^e)(1 + i_{1,t+2}^e) = (1 + i_{3,t} - tp_{3,t})(1 + i_{3,t} - tp_{3,t})(1 + i_{3,t} - tp_{3,t})$$

- Solving for $i_{1,t+2}^e$ we have:

$$\begin{aligned} i_{1,t+2}^e &= \frac{(1 + i_{3,t} - tp_{3,t})(1 + i_{3,t} - tp_{3,t})(1 + i_{3,t} - tp_{3,t})}{(1 + i_{1,t})(1 + i_{1,t+1}^e)} - 1 \\ &= \frac{(1 + 6\% - 0.5\%)(1 + 6\% - 0.5\%)(1 + 6\% - 0.5\%)}{(1 + 5\%)(1 + 5.5\%)} - 1 = 6\% \end{aligned}$$