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Workshop to Identify New Sources of Growth

Fundamentals of Economic Growth

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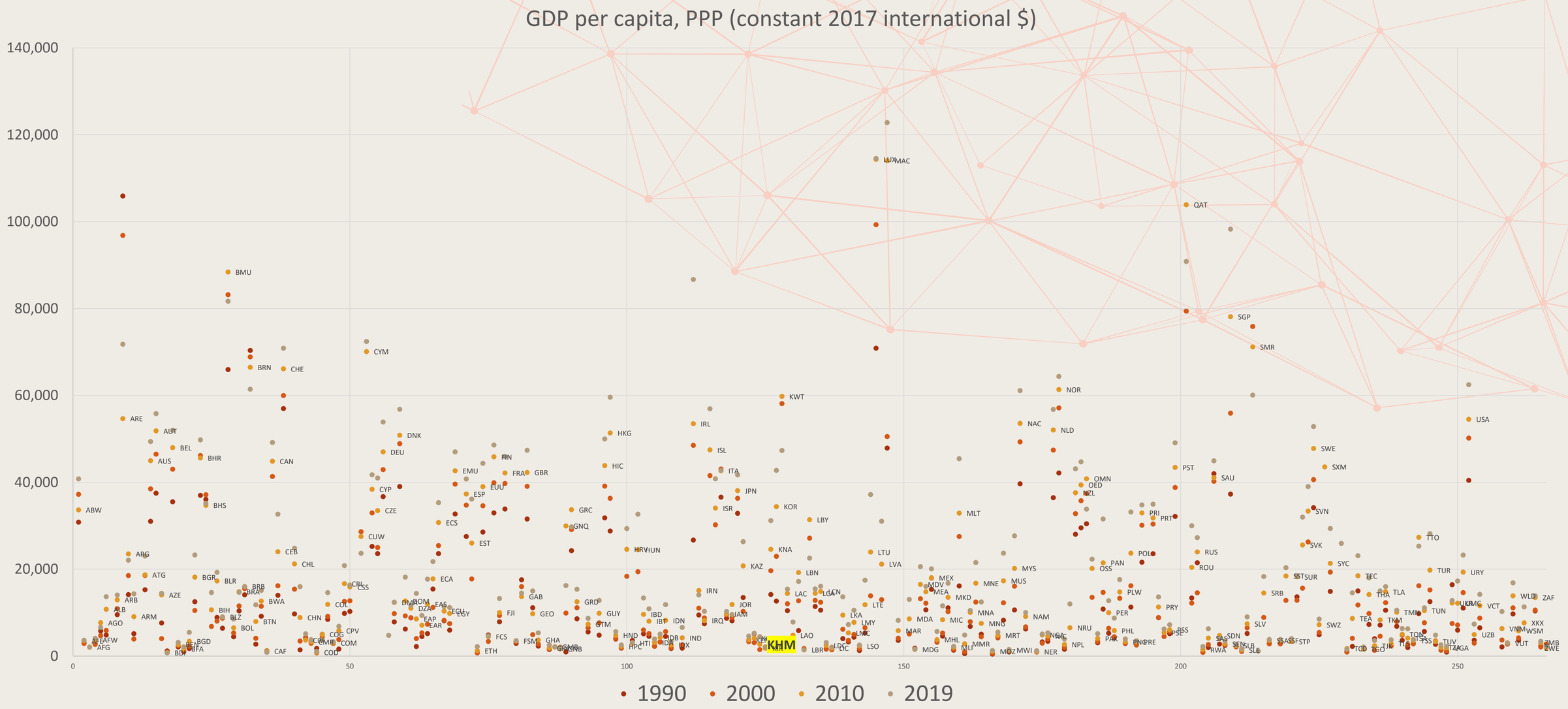
July 13, 2023.

Fundamentals of Economic Growth

1. **Main Facts of Economic Growth**
   i. Charles Jones and Dietz Vollrath (2013), "Introduction to Economic Growth," 3rd edition, WW Norton.
   ii. Daron Acemoglu (2008), "Introduction to Modern Economic Growth," Princeton University Press.
   iii. William Easterly (2002), "The Elusive Quest for Growth: Economists' Adventures and Misadventures in the Tropics," MIT Press.
2. **Solow Growth Model**
   i. Robert Solow (1956), "The Theory of Economic Growth," Quarterly Journal of Economics, 70 (1): 65-94.
3. **Extending the Solow Growth Model with Human Capital**
   i. Gregory Mankiw, David Romer and David Weil (1992), "A Contribution to the Empirics of Economic Growth," Quarterly Journal of Economics, 107 May :407-437.
4. **Technological Growth and Endogenous Growth**
   i. Paul Romer (1994), "The Origins of Endogenous Growth," Journal of Economic Perspectives, 8 (1): 3-22.
5. **Creative Destruction: A Schumpeterian Model**
   i. Ricardo Caballero (2008), "Creative Destruction," in The New Palgrave Dictionary of Economics.
6. **Institutions**
   i. Robert Hall and Charles Jones (1999), "Why Do Some Countries Produce So Much More Output Per Worker Than Others?," The Quarterly Journal of Economics, 114 (1): 83-116.
   ii. Daron Acemoglu, Simon Johnson and James Robinson (2001), "The Colonial Origins of Comparative Development: An Empirical Investigation," The American Economic Review, 91 (5): 1369 -1401.
7. **Newer Ideas on Economic Growth**
   i. Charles Jones and Paul Romer (2010), "The New Kaldor Facts: Ideas, Institutions, Population, and Human Capital," American Economic Journal: Macroeconomics, 2 (1): 224-245.
   ii. Charles Jones (2016), "The Facts of Economic Growth," Handbook of Macroeconomics, Volume 2A, Elsevier B. V. Chapter 1: 3-69.
   iii. Ricardo Hausmann, Dani Rodrik and Andrés Velasco (2005), "Growth Diagnostics," CID, Harvard University.

Standard of living across the world



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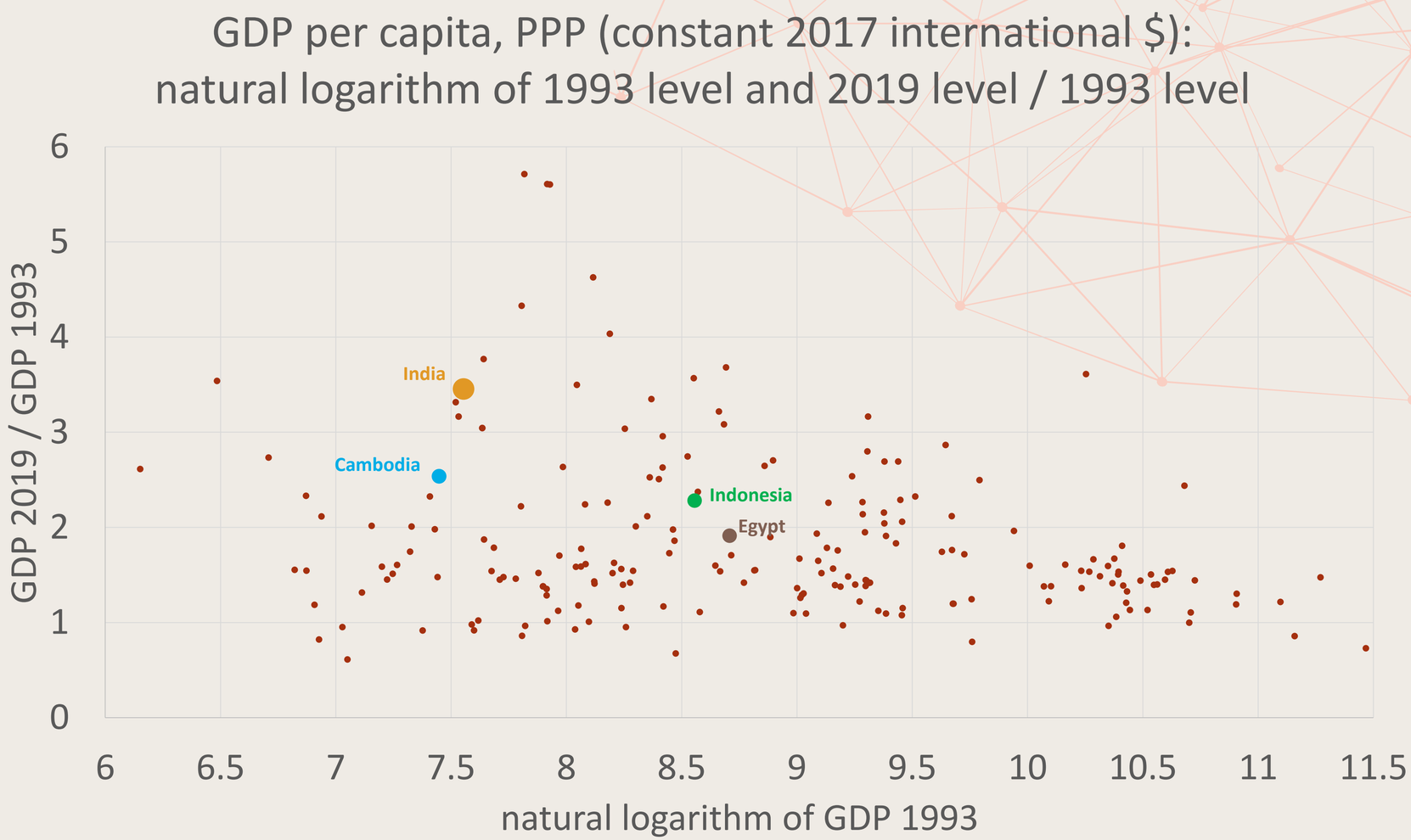
Facts on the evolution of standard of living

- **Fact 1**: There is enormous variation in per capita income across economies. The poorest countries have per capita incomes that are less than 5 percent of per capita income in the richest countries.
- **Fact 2**: Rates of economic growth vary substantially across countries.
- **Fact 3**: Growth rates are not generally constant over time. For the world as a whole, growth rates were close to zero over most of history but have increased sharply in the twentieth century. For individual countries, growth rates also change over time.
- **Fact 4**: A country's relative position in the world distribution of per capita incomes is not immutable. Countries can go from being "poor" to being "rich", and vice versa.
- **Fact 5**: In the U.S. over the last century,
  - The real rate of return on capital, r, shows no trend upward or downward;
  - The shares of income devoted to capital, rK/Y, and labor, wL/Y, show no trend; and
  - The average growth rate of output per person has been positive and relatively constant over time - that is, the United States exhibits steady, sustained per capita income growth.
- **Fact 6**: Growth in output and growth in the volume of international trade are closely related.
- **Fact 7**: Both skilled and unskilled workers tend to migrate from poor to rich countries or regions.

Standard of living across the world



Big questions:

- Why are some countries so rich and others so poor?
- What are the engines for growth?
- What creates growth miracles?

"I do not see how one can look at figures like these without seeing them as representing possibilities. Is there some action a government of India could take that would lead the Indian economy to grow like Indonesia's or Egypt's? If so, what, exactly? If not, what is it about the "nature of India" that makes it so? The consequences for human welfare involved in questions like these are simply staggering: Once one starts to think about them, it is hard to think about anything else." Robert E. Lucas Jr. (1988), On the mechanics of economic development, Journal of Monetary Economics 22.

0. Objective

- Understand the long-run determinants of living standards.

- Study the transition dynamics of an economy within the context of a neoclassical framework.

- Quantify the sources of growth in relatively simple growth accounting framework.

1. Cross-Country Income Difference

There are very large differences in income per capita and output per worker across countries today.

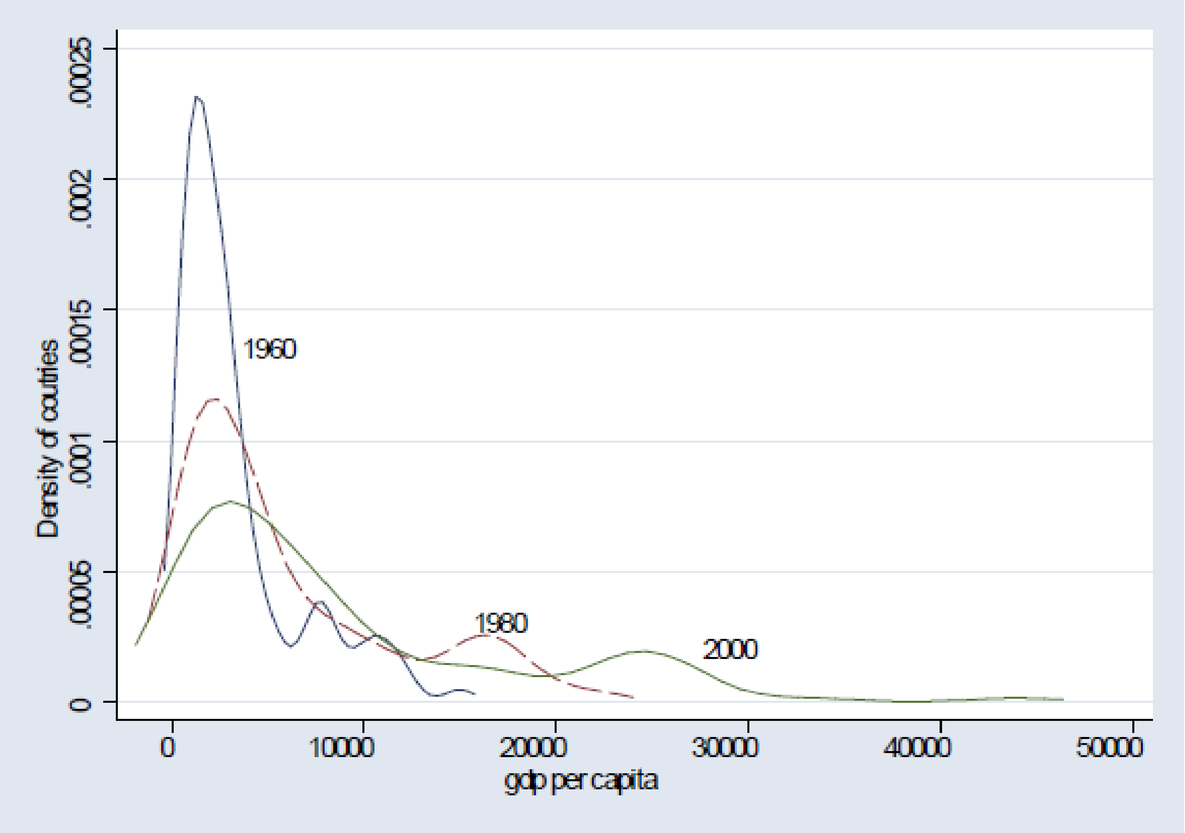


Figure: Distribution of PPP-adjusted GDP per capita.

1. Cross-Country Income Difference (cont.)

Rich countries

Country	GDP per capita 2008	GDP per worker 2008	LF Part. Rate 2008	Avg. Growth 1960-2008	Years to Double
United States	$43,326	$84,771	0.51	1.6	43
Japan	33,735	64,778	0.52	3.4	21
France	31,980	69,910	0.46	2.2	30
United Kingdom	35,345	70,008	0.51	1.9	36
Spain	28,958	57,786	0.50	2.7	26

Poor countries

Country	GDP per capita 2008	GDP per worker 2008	LF Part. Rate 2008	Avg. Growth 1960-2008	Years to Double
China	6,415	10,938	0.59	5.6	13
India	3,078	7,801	0.39	3.0	24
Nigeria	1,963	6,106	0.32	0.6	114
Uganda	1,122	2,604	0.43	1.3	52

Growth miracles

Country	GDP per capita 2008	GDP per worker 2008	LF Part. Rate 2008	Avg. Growth 1960-2008	Years to Double
Hong Kong	37,834	70,940	0.53	4.3	16
Singapore	49,987	92,634	0.54	4.1	17
Taiwan	29,645	62,610	0.47	5.1	14
South Korea	25,539	50,988	0.50	4.5	16

Growth disasters

Country	GDP per capita 2008	GDP per worker 2008	LF Part. Rate 2008	Avg. Growth 1960-2008	Years to Double
Venezuela	9,762	21,439	0.46	-0.1	-627
Haiti	1,403	3,164	0.44	-0.4	-168
Madagascar	810	1,656	0.49	-0.1	-488
Zimbabwe	135	343	0.40	-1.5	-47

Cross-Country Income Difference

- Part of the spreading out of the distribution in the Figure is because of the increase in average incomes.

- More natural to look at the log of income per capita when growth is approximately proportional:
  - when x (t) grows at a proportional rate, log x (t) grows linearly
  - if x1 (t) and x2 (t) both grow by 10%, x1 (t) - x2 (t) will also grow, while log x1 (t) - log x2 (t) will remain constant.

- The next Figure shows a similar pattern, but now the spreading-out is more limited.

Cross-Country Income Difference

Figure: Estimates of the distribution of countries according to log GDP per capita (PPP-adjusted) in 1960, 1980 and 2000.

Cross-Country Income Difference

Figure: Estimates of the distribution of countries according to log GDP per worker (PPP-adjusted) in 1960, 1980 and 2000.

2. Why growth matters?

- Anything that affects the long-run rate of economic growth – even by a tiny amount – will have huge effects on living standards in the long run.

Annual growth rate of income per capita	Percentage increase in standard of living after ___ years		
	25	50	100
2.0%	64.1	169.2	624.5
2.5%	85.4	243.7	1,081.4
5.0%	238.6	1,046.7	13,050.1
7.0%	442.7	2,845.7	86,671.6

Income Growth and Income Level

- Higher income growth is associated with higher income levels

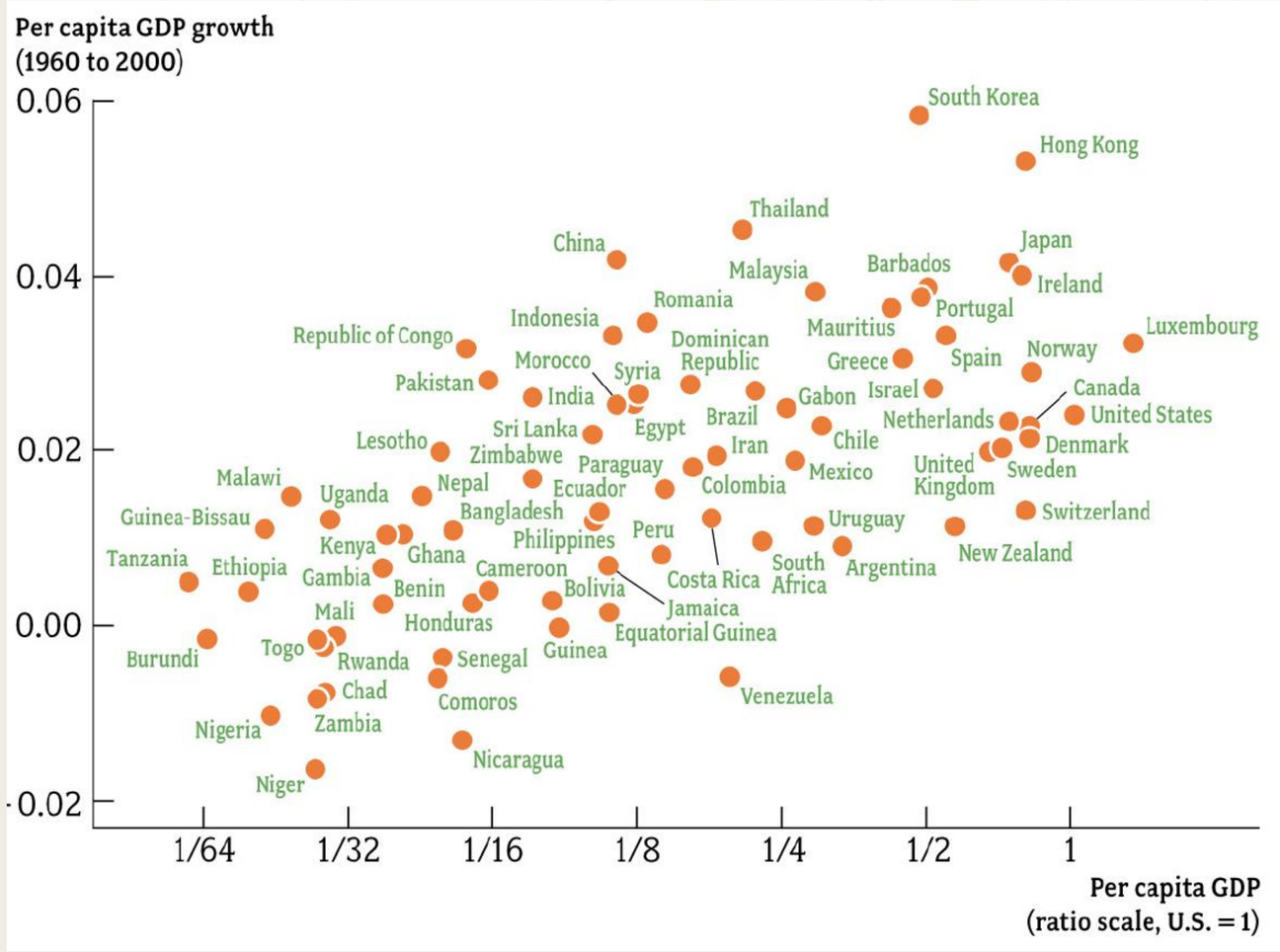
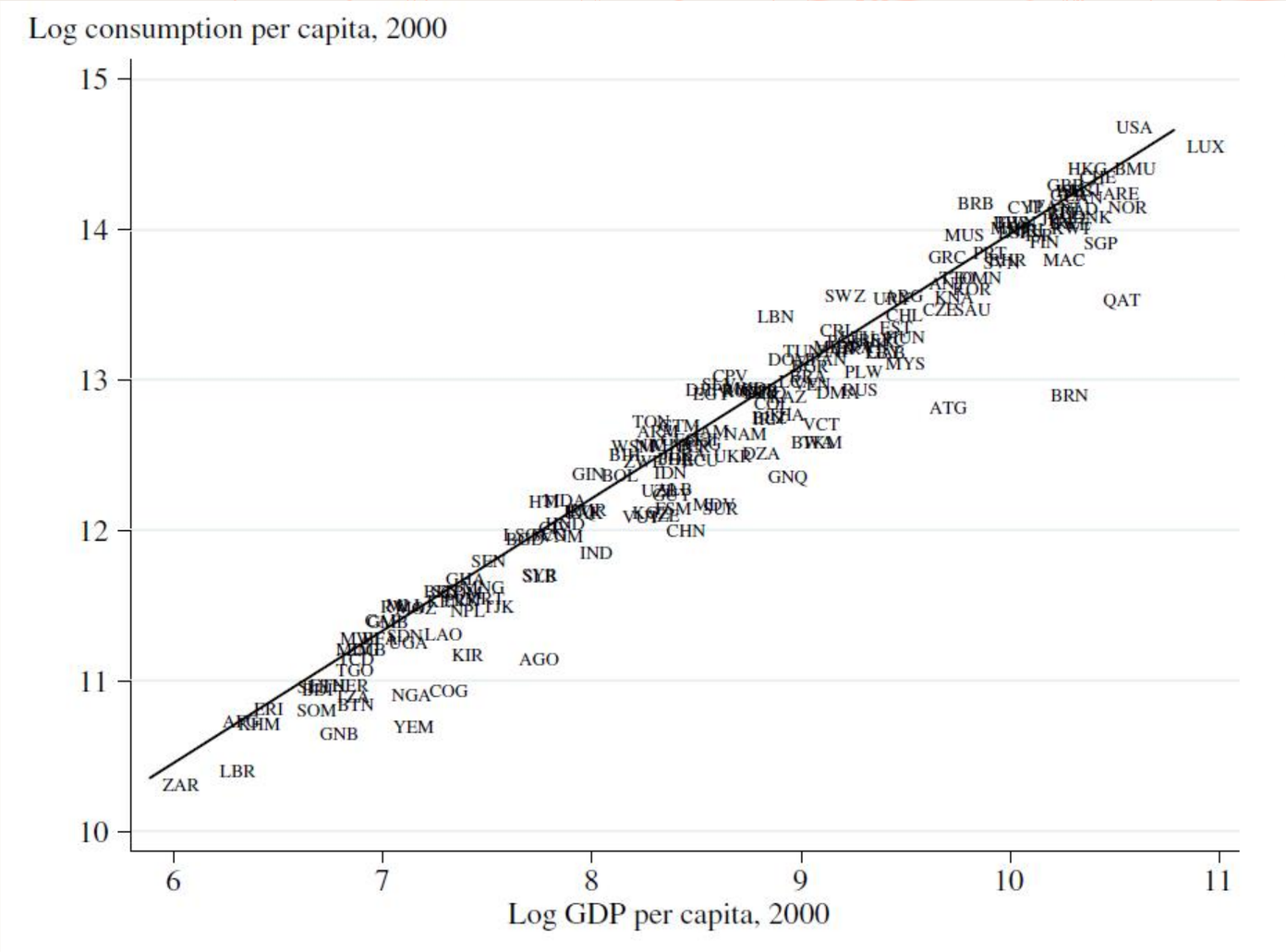


Figure Levels and Growth Rates of Per Capita GDP

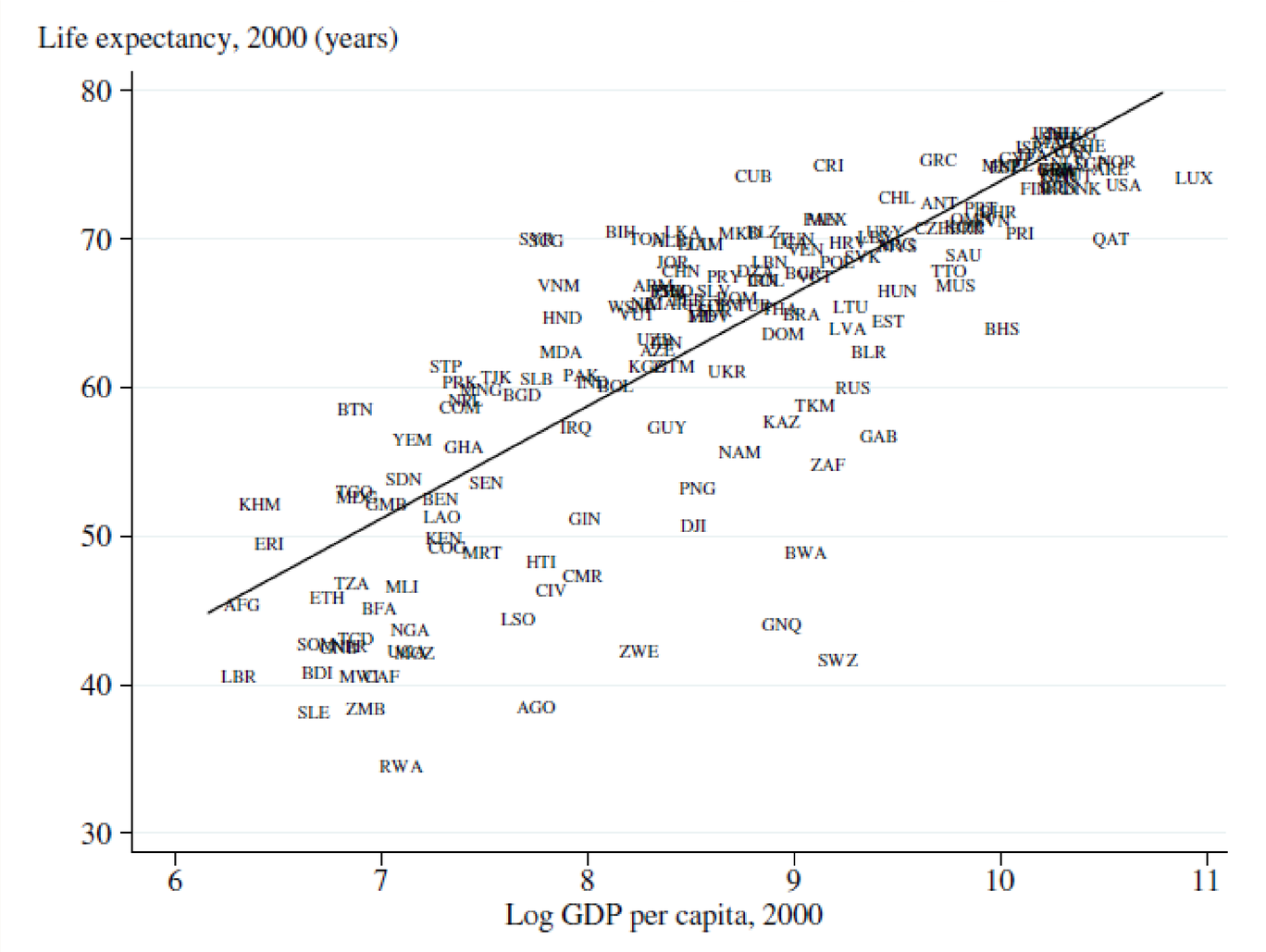
Income Level and Consumption

- and higher income supports higher consumption



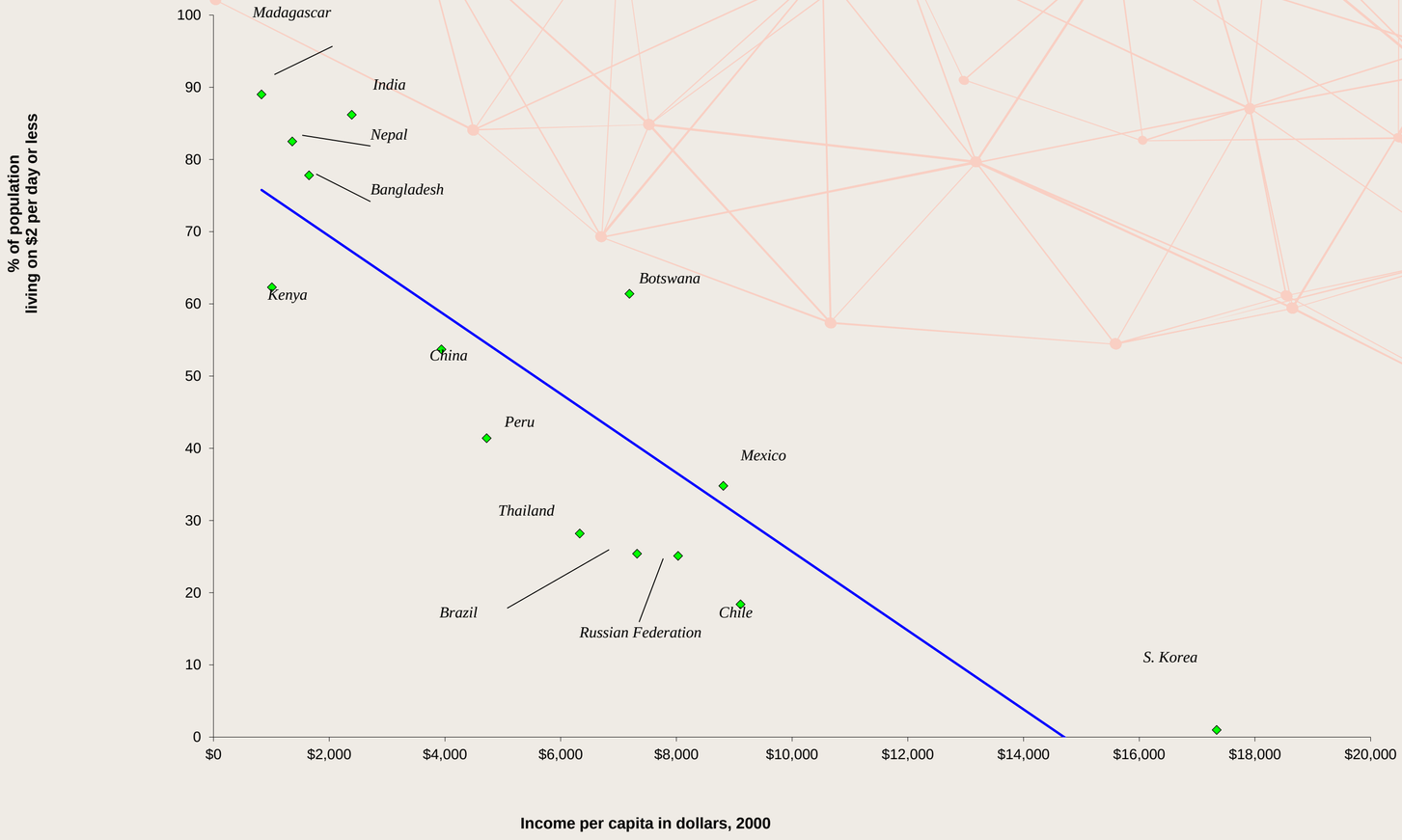
Income Level and Life Expectancy

- is associated with longer life expectancy



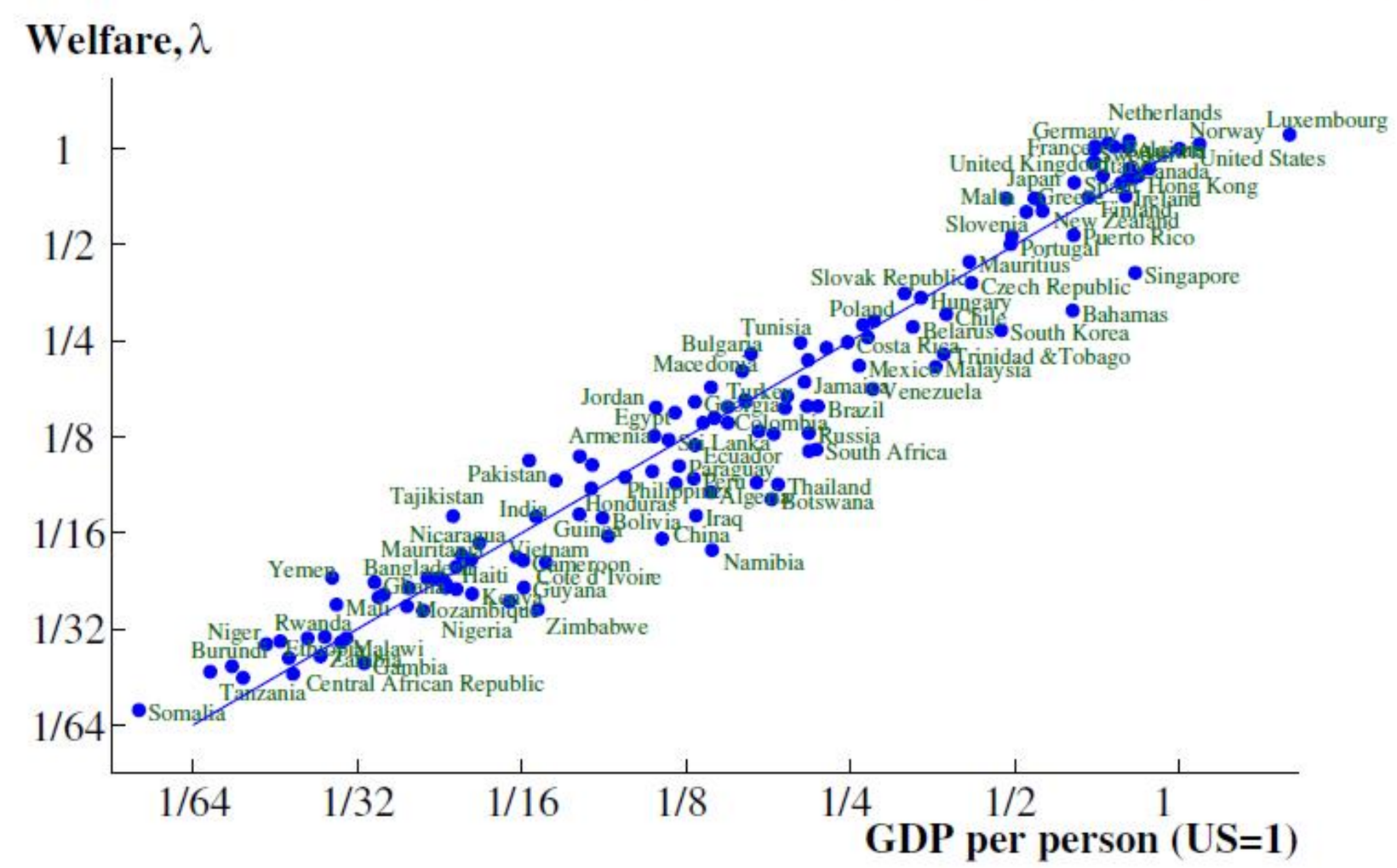
Income Level and Poverty

- and lowers poverty



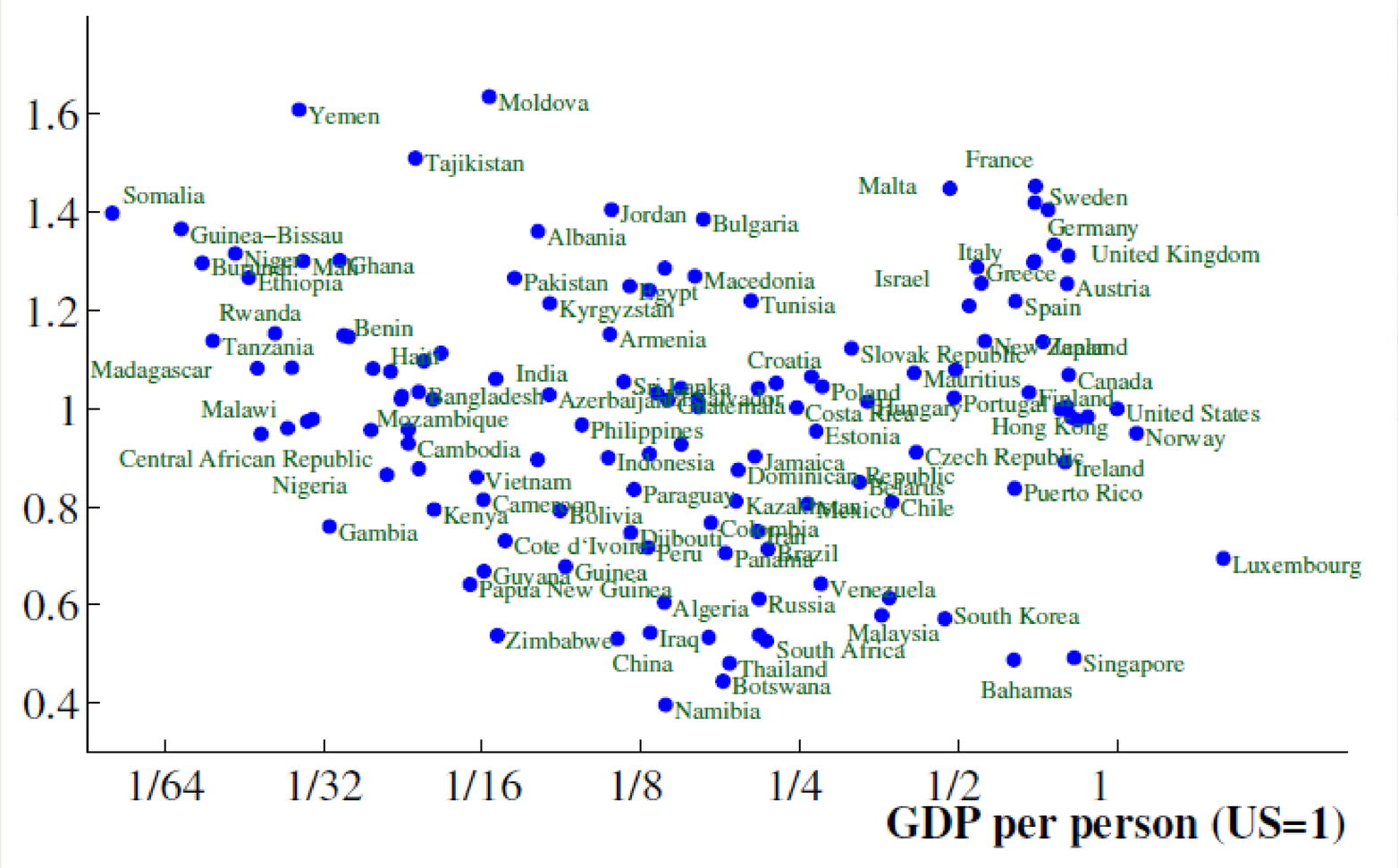
Welfare vs. GDP (Jones and Klenow, 2010)

Welfare combines consumption, leisure, inequality, life expectancy



Ratio of Welfare to Income

But the deviations of welfare from GDP are large



3. Why are income levels and growth rates so different?

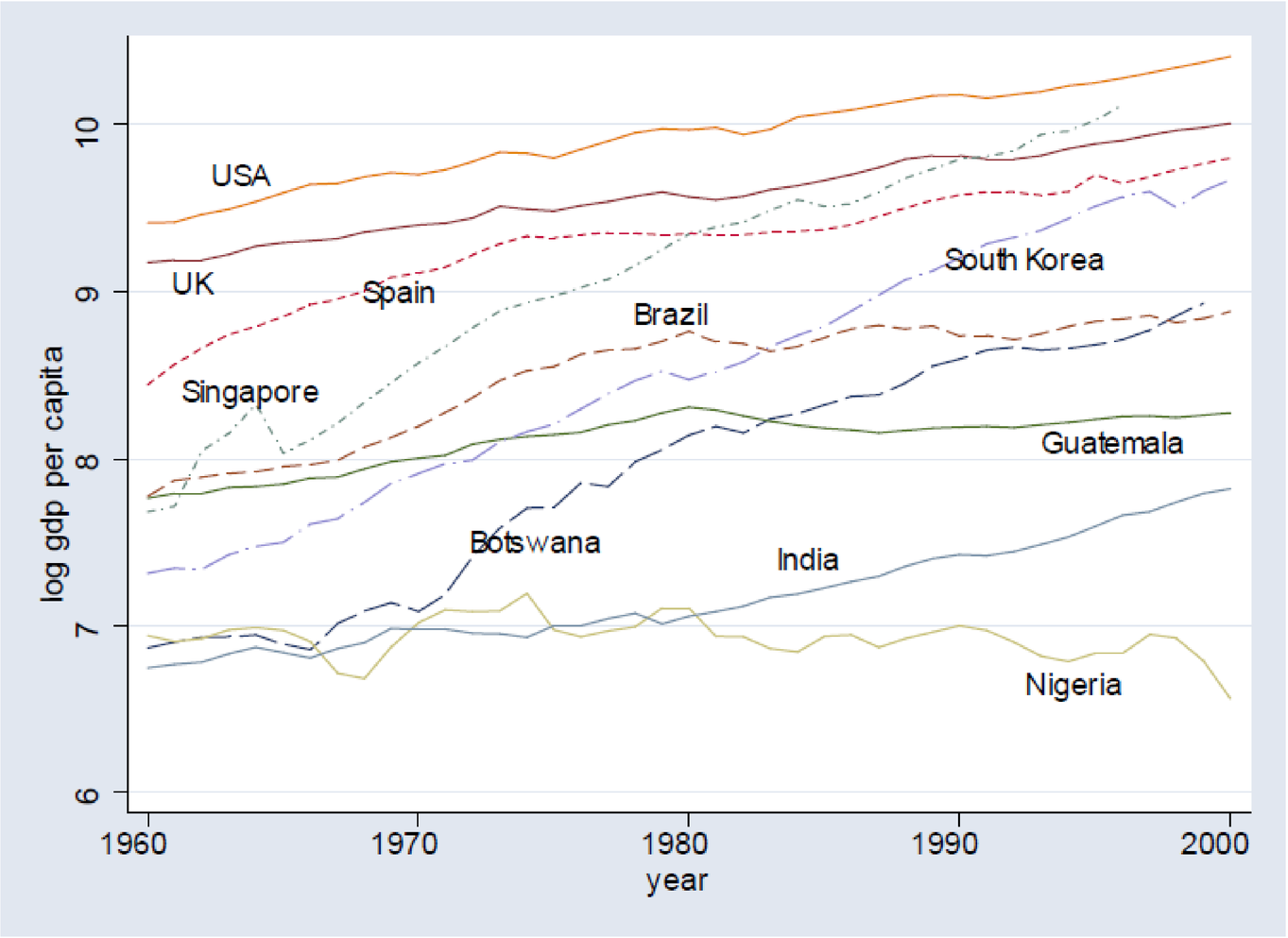


Figure The Evolution of Income Per Capita GDP

Why are income levels and growth rates so different?

- Why is the United States richer in 1960 than other nations and able to grow at a steady pace thereafter?

- How did Singapore, South Korea and Botswana manage to grow at a relatively rapid pace for 40 years?

- Why did Spain grow relatively rapidly for about 20 years, but then slow down? Why did Brazil and Guatemala stagnate during the 1980s?

- What is responsible for the disastrous growth performance of Nigeria?

- Our first task is to develop a coherent framework to investigate these questions and as a byproduct we will introduce the workhorse models of dynamic economic analysis and macroeconomics.

Origins of Income Differences and World Growth

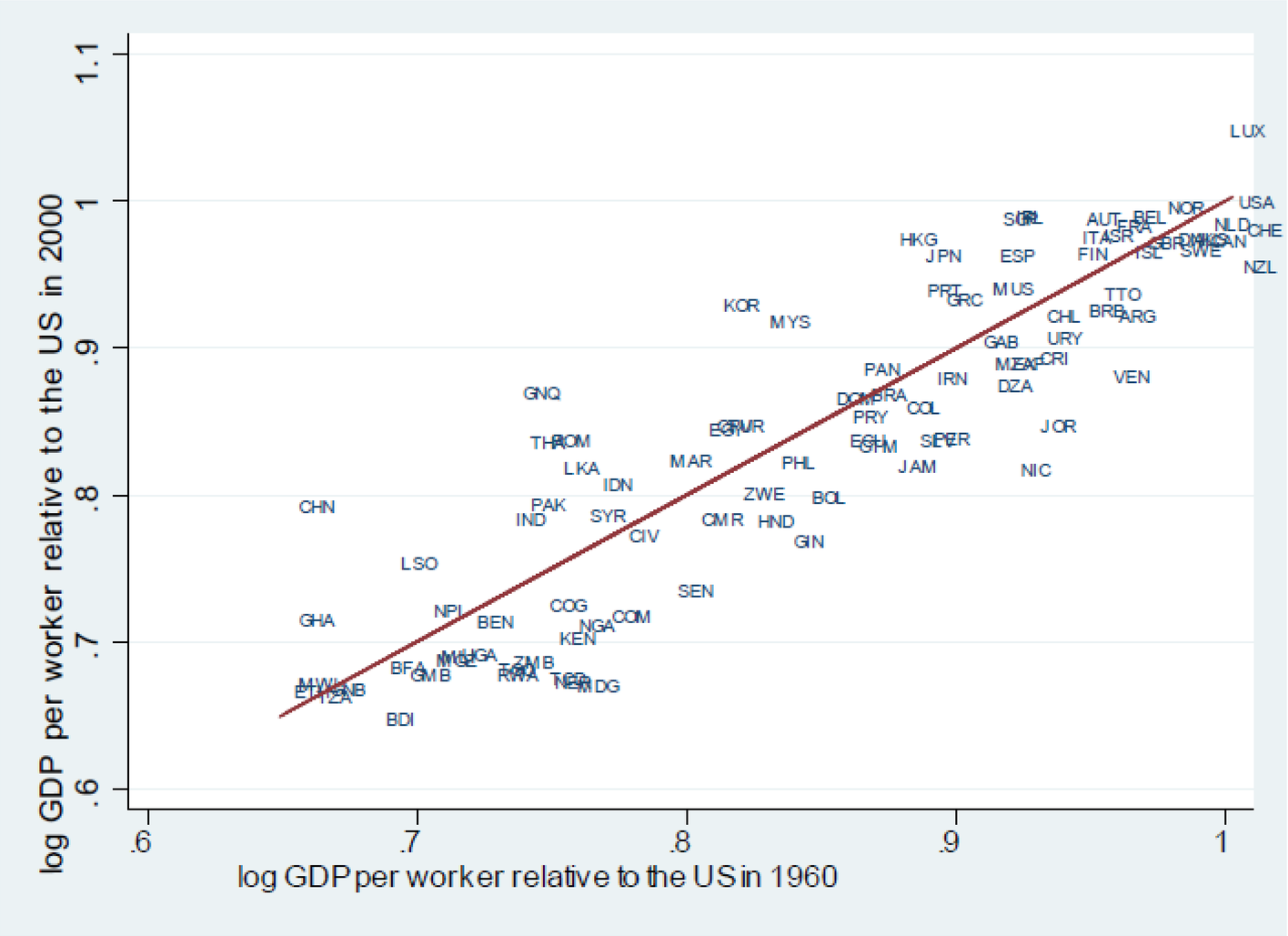


Figure Log GDP per worker in 1960 and 2000

A long view

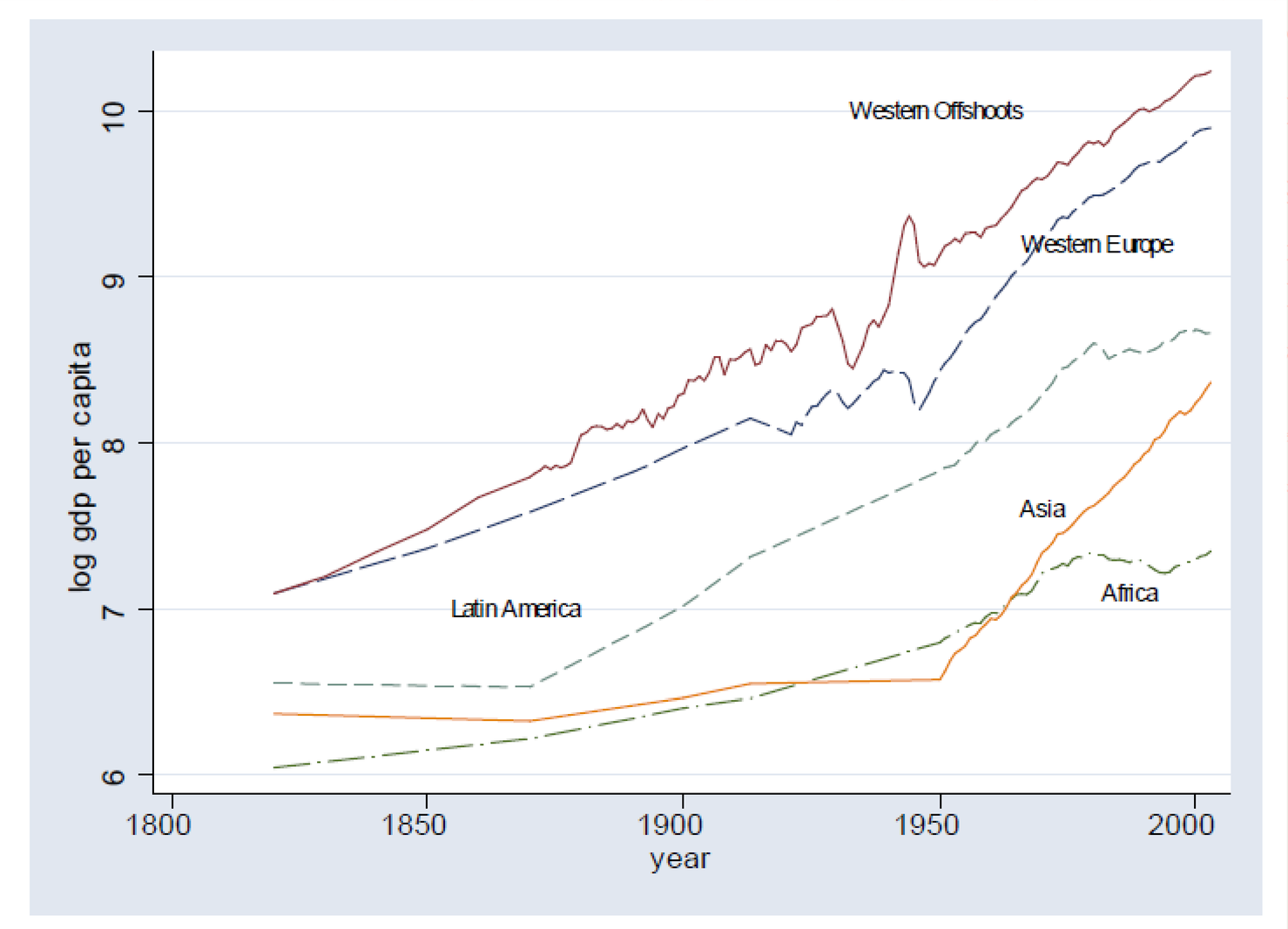


Figure Evolution of GDP per Capita 1820 - 2000

An even longer view

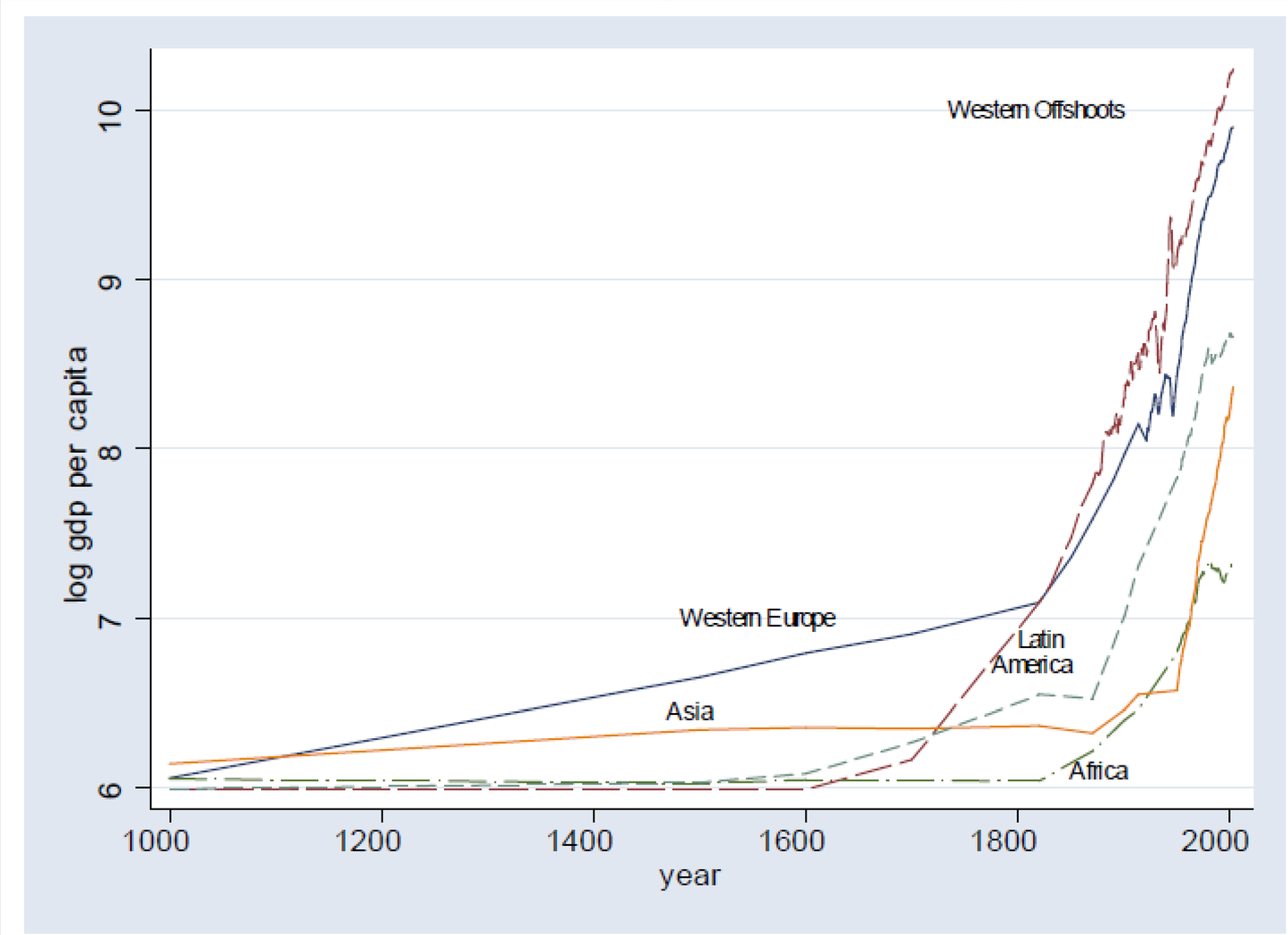


Figure Evolution of GDP per Capita 1000 - 2000

The Elusive Quest for Growth

- In the book The Elusive Quest for Growth: Economists' Adventures and Misadventures in the Tropics, William Easterly explains how in the aftermath of World War II, economists began the quest to discover the means by which poor countries in the tropics could become rich like rich countries in Europe and North America.
- Different treatments including providing foreign aid to investment in machines, fostering education, controlling population growth, giving loans conditional on reforms, and giving debt relief conditional on reforms have had mixed effects to achieve the expected growth.
- The problem was not the failure of economics, but the failure to apply the principles of economics in practical policy work: “people respond to incentives; all the rest is commentary” Steven Landsburg The Armchair Economist.
- The development process is conditioned by incentives of donors, governments, and citizens, that form a complicated web that is not easily untangled.

The Elusive Quest for Growth (cont.)

- The book is divided in 3 sections and 14 chapters.
- Section **I Why Growth Matters** has the chapter 1 To Help the Poor.
- Section **II Panaceas that Failed** has the chapters 2 Aid for Investment, 3 Solow's Surprise: Investment Is Not the Key to Growth, 4 Educated for What?, 5 Cash for Condoms?, 6 The Loans That Were, the Growth That Wasn't, and 7 Forgive Us Our Debts.
- Section **III People Respond to Incentives** has the chapters 8 Tales of Increasing Returns: Leaks, Matches, and Traps, 9 Creative Destruction: The Power of Technology, 10 Under an Evil Star, 11 Governments Can Kill Growth, 12 Corruption and Growth, 13 Polarized Peoples, and 14 Conclusion: The View from Lahore.
- In between each chapter there are very insightful and entertaining snapshots of daily life in the Third World, "intermezzos", that reminds us that behind the quest for growth are the suffering and joys of real people, and it is for them we go on the quest for growth.

4. The Role of Models

If we understand the process of economic growth — or of anything else — we ought to be capable of demonstrating this knowledge by creating it in these pen and paper (and computer-equipped) laboratories of ours. If we know what an economic miracle is, we ought to be able to make one. — Robert E. Lucas, Jr.

Is it appropriate to use toy models to study the world?

- All models make simplifying assumptions
- What makes a model successful is making the correct simplifying assumptions.
- Healthy skepticism is warranted. Even when model gets it wrong, the errors may be informative...

Solow Growth Model

- Develop a simple framework for the proximate causes and the mechanics of economic growth and cross-country income differences.

- Solow-Swan model named after Robert Solow and Trevor Swan

- Study of economic growth and development therefore necessitates dynamic models.

- Despite its simplicity, the Solow growth model is a dynamic general equilibrium model (though many key features of dynamic general equilibrium models, such as preferences and dynamic optimization are missing in this model)

Solow Growth Model

Production

Real output, Y, is produced according to a function like this:

$$Y = F(K, L) \tag{1}$$

where K is the stock of physical capital and L is the number of workers.

It is common to use a very specific function, $F(\cdot)$, called the Cobb-Douglas

$$Y = K^{\alpha} L^{1-\alpha} \tag{2}$$

This function has constant returns to scale, which means that if you scale each input by a factor z then output changes by the same proportion:

$$(zK)^{\alpha}(zL)^{1-\alpha} = zK^{\alpha}L^{1-\alpha} = zY \tag{3}$$

Solow Growth Model

Firms

We assume that firms are perfectly competitive

- There are many firms, all producing the same homogeneous output
- Firms enter and exit freely
- They all produce using a similar Cobb-Douglass function
- They are all price-takers for the use of labor and capital

For the representative firm, profits are

$$\Pi = K^{\alpha} L^{1-\alpha} - rK - wL \tag{4}$$

where r is the market rate for renting capital and w is the wage of a worker.

The firm's profit-maximizing decision (first-order conditions) are

$$\mathrm{MPK} = \alpha \frac{Y}{K} = r \tag{5}$$

$$\mathrm{MPL} = (1-\alpha) \frac{Y}{L} = w \tag{6}$$

Which just say that the firm sets the marginal product of a factor equal to its marginal cost.

Factor incomes are $rK = \alpha Y$ and $wL = (1-\alpha)Y$, while profits are zero, $\Pi = 0$.

Factor shares are $\frac{rK}{Y} = \alpha$ and $\frac{wL}{Y} = 1 - \alpha$.

Historical Factor Shares

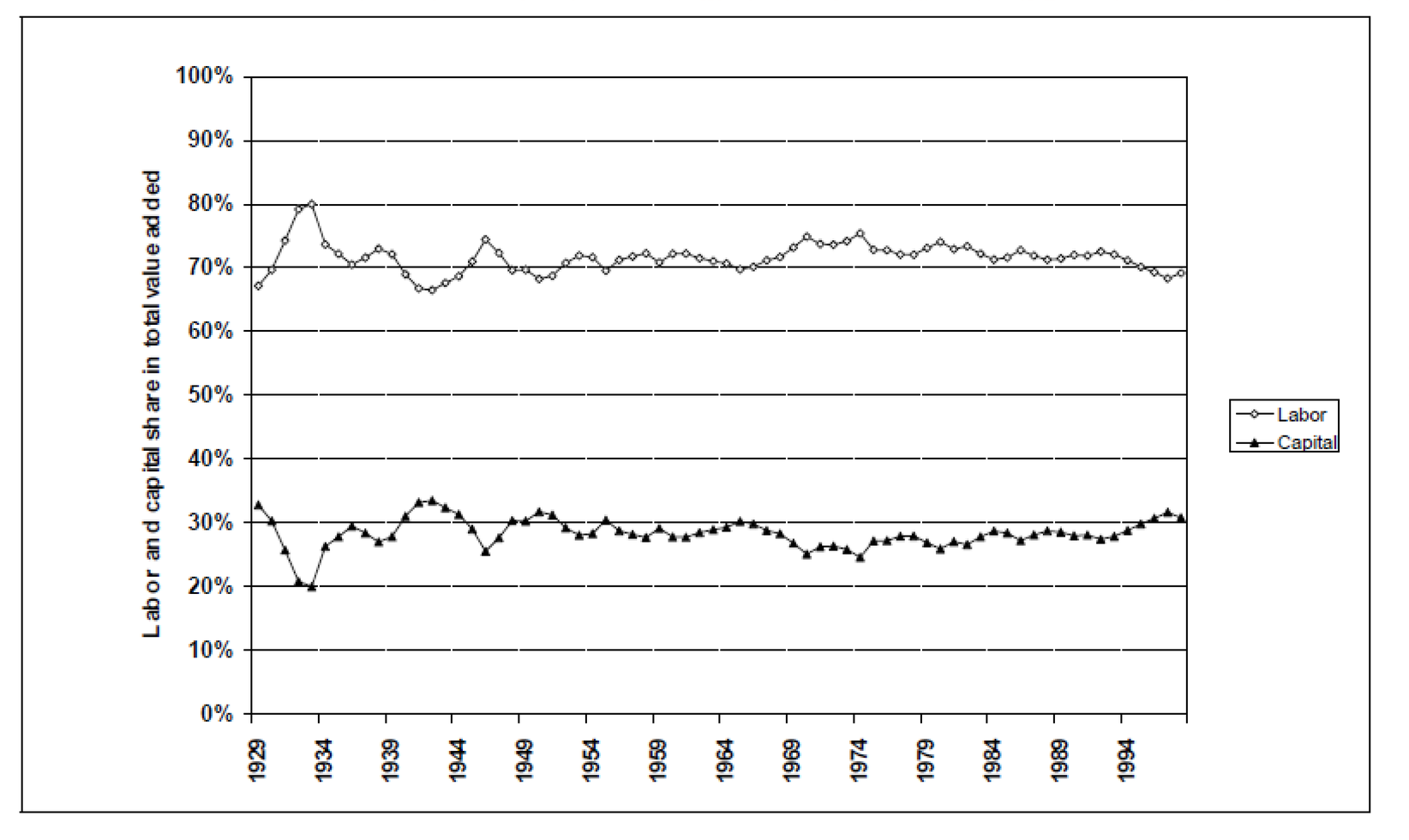


Figure: Capital (rK/Y) and Labor (wL/Y) share in the U.S. GDP

Solow Growth Model

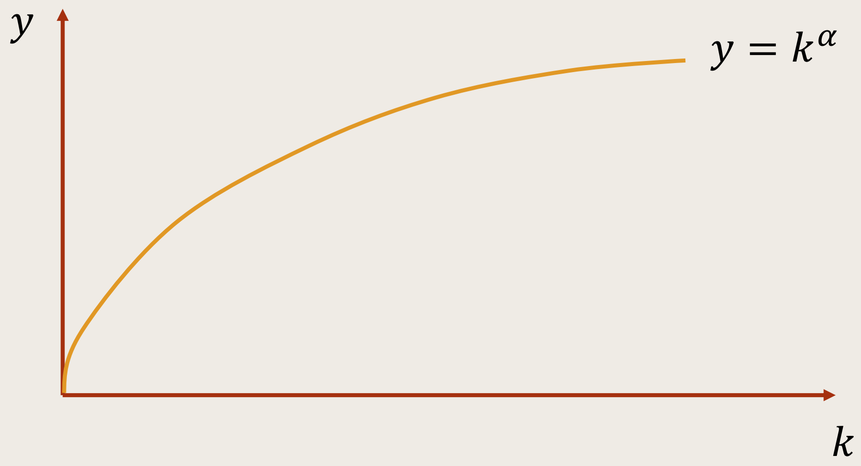
Output per worker

We typically care about output per worker

$$y = \frac{Y}{L} = \frac{K^{\alpha}L^{1-\alpha}}{L} = \frac{K^{\alpha}}{L^{\alpha}} = k^{\alpha} \qquad (7)$$

where we are using $y = \frac{Y}{L}$ and $k = \frac{K}{L}$ to represent per-worker values.

Capital per worker has a diminishing marginal output. If k rises, output per worker rises, but the size of the increase falls as k increases.



Solow Growth Model

Capital Accumulation

Given that output depends on capital and labor, we need to know who those two things accumulate over time. Start with capital, where its change K in continuous time or $K_{t+1} - K_t$ in discrete time, is given by

$$\dot{K} = sY - \delta K \tag{8}$$

where sY is gross investment, where individuals who work and own capital – save a constant fraction, $0 < s < 1$, of their income. Meanwhile, δK is depreciation where a fixed fraction, $0 < \delta < 1$, of capital breaks down.

Dividing (8) by K we have that the growth rate of capital

$$\frac{\dot{K}}{K} = s\frac{Y}{K} - \delta \tag{9}$$

To see what happens to capital per worker, $k = \frac{K}{L}$, take logs, $\ln k = \ln K - \ln L$, and derivatives $\frac{\partial}{\partial t}\ln k = \frac{\partial}{\partial t}\ln K - \frac{\partial}{\partial t}\ln L$, or $\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L}$. Replacing equation (9) in this last expression, we get

$$\frac{\dot{k}}{k} = s\frac{Y}{K} - \delta - \frac{\dot{L}}{L} \tag{10}$$

Solow Growth Model

Population Growth

We assume that the number of workers grows at the same rate as the population. Let the number of workers exhibit exponential growth according to

$$L(t) = L(0)e^{nt} \tag{11}$$

By taking logs and derivatives to equation (11) we can see that the growth rate of the number of workers is given by

$$\frac{\dot{L}}{L} = n \tag{12}$$

Therefore, replacing (12) into (10), we have

$$\frac{\dot{k}}{k} = s\frac{Y}{K} - \delta - n \tag{13}$$

Therefore, the growth rate of capital per worker depends on the capital/output ratio $\frac{Y}{K}$ and the parameters s, δ and n.

Solow Growth Model

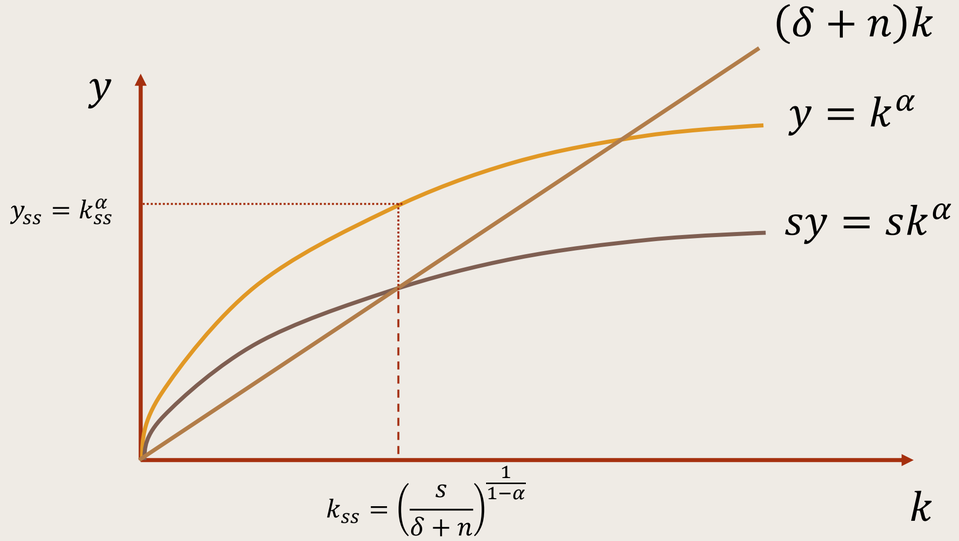
The Solow Equation

Equation (13) can be rewritten as

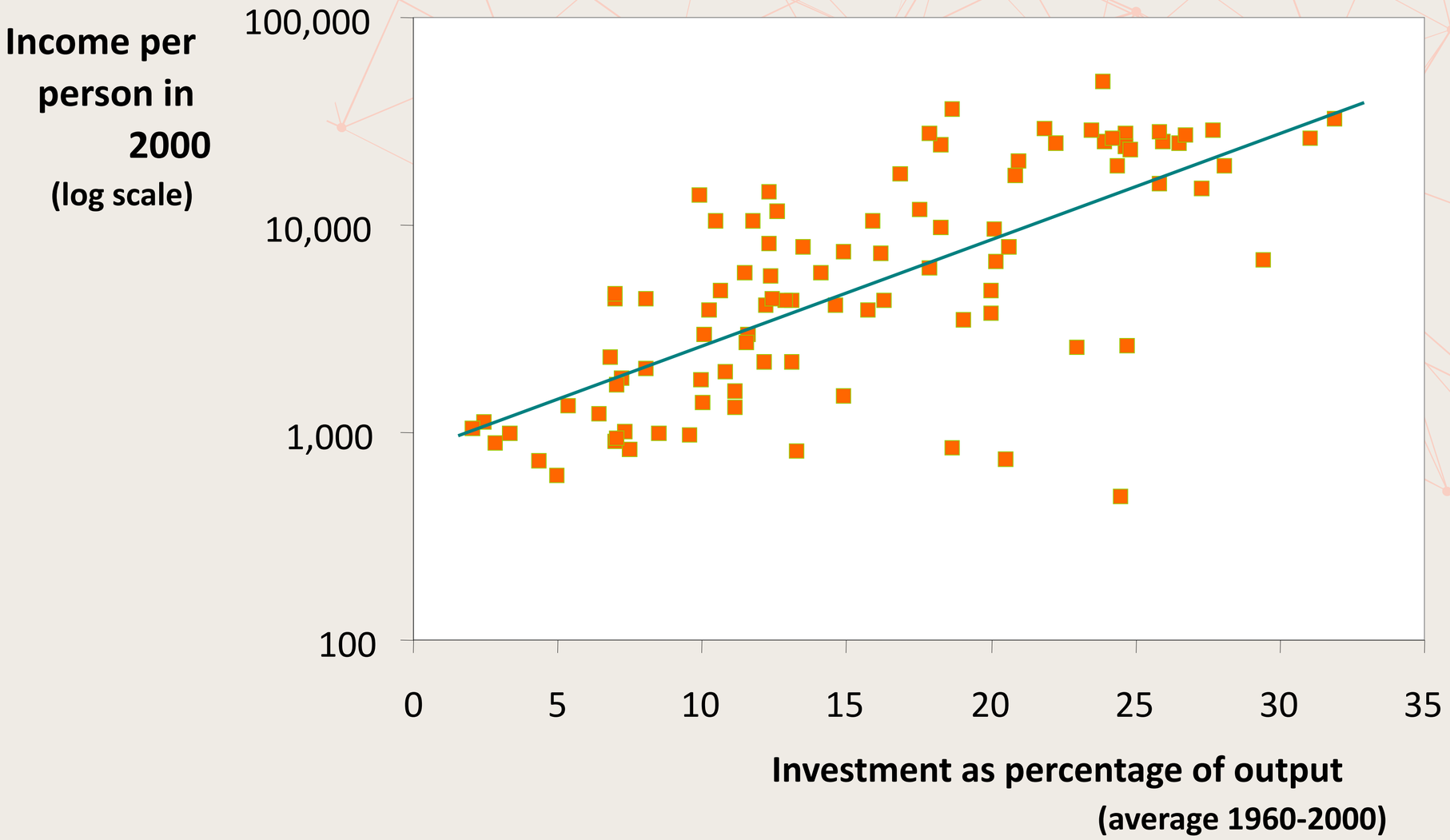
$$\frac{\dot{k}}{k} = s\frac{Y/L}{K/L} - \delta - n = s\frac{y}{k} - \delta - n \qquad (14)$$

Multiplying both sides of equation (14) by k and recalling from (7) that $y = k^{\alpha}$, we have

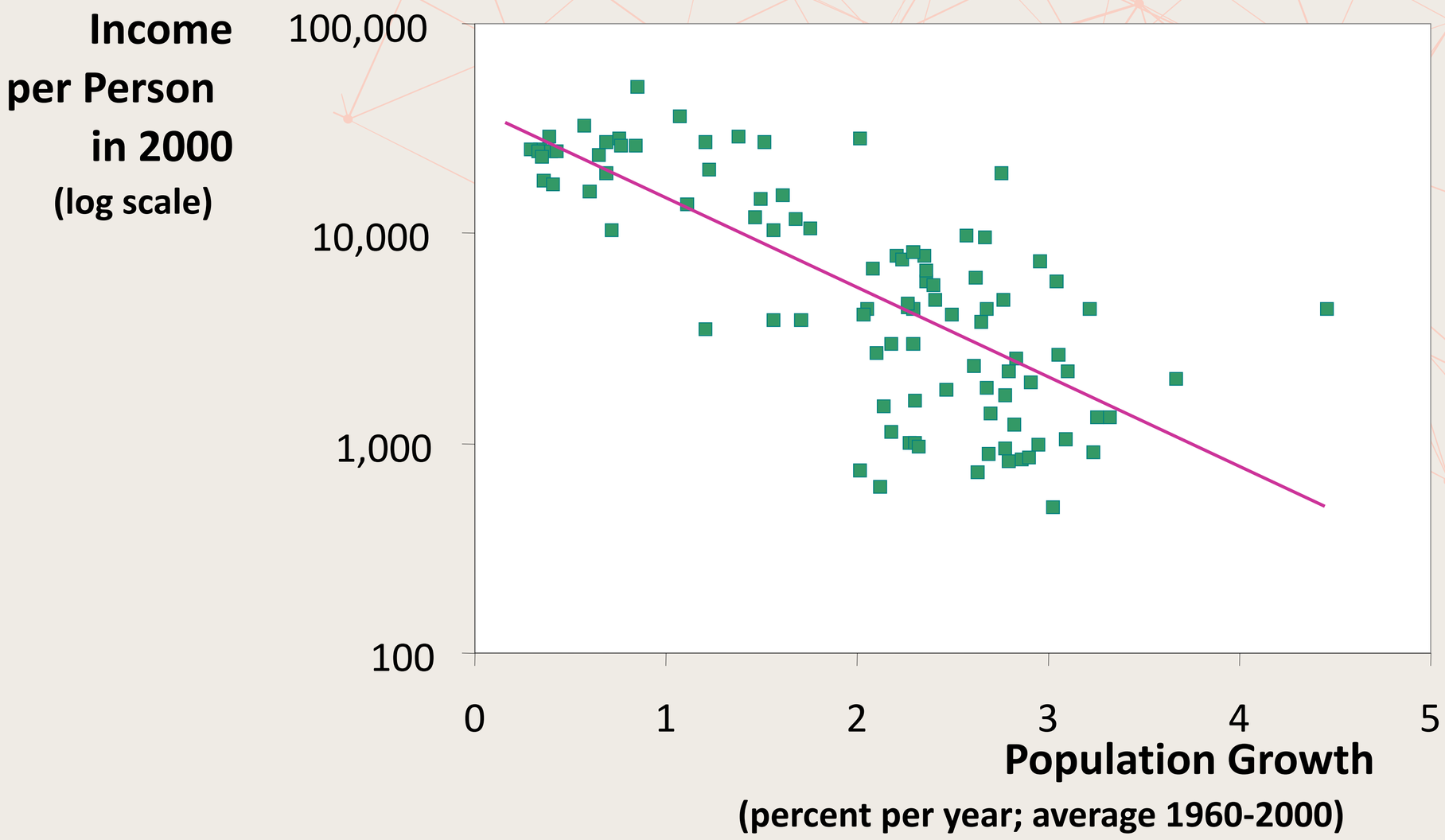
$$\dot{k} = sk^{\alpha} - (\delta + n)k \qquad (15)$$



International evidence on investment rates and income per person



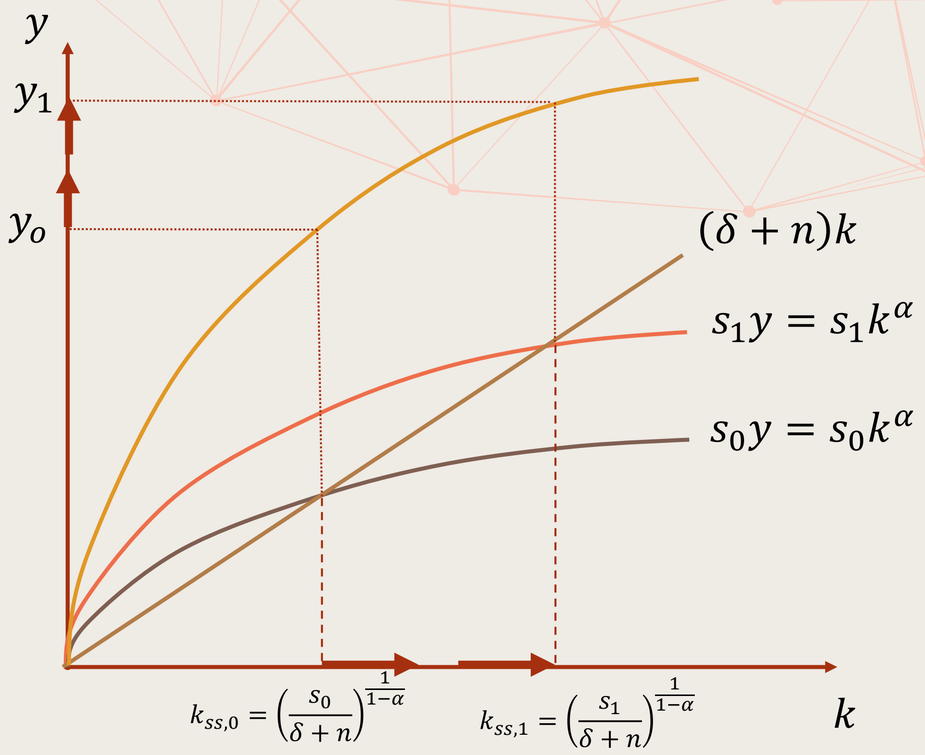
International evidence on population growth and income per person



Solow Growth Model

Comparative statics

What if the savings rate, s, changes? This could be due to a policy change, or some difference in individuals' willingness to save for the future. Let assume that it increases to $s_1 > s_0$.



Correlates of Economic Growth

Figure: Average Investment to GDP Ratio and Economic Growth

Solow Growth Model

Trend Growth

In this simple setup of the Solow model, growth rate goes to zero, y does not grow in steady state. To see this, take logs and derivatives of $y = k^{\alpha}$ to find $\ln y = \alpha \ln k$ and $\frac{\dot{y}}{y} = \alpha \frac{\dot{k}}{k}$. Because $\dot{k} = 0$ in steady state, it must be that $\frac{\dot{y}}{y} = 0$ in steady state.

To allow for trend growth in the Solow model, we will incorporate productivity improvements. Let production be

$$Y = K^{\alpha}(AL)^{1-\alpha} \tag{16}$$

where A is labor-augmenting technological progress. In per worker terms this is

$$y = k^{\alpha}A^{1-\alpha} \tag{17}$$

Growth in A will allow there to be trend growth in output per worker even in steady state. If we let that $\mathrm{A}(t) = A(0)e^{gt}$, then by taking logs and derivatives $\frac{\dot{A}}{A} = g$.

Solow Growth Model

Trend Growth (cont.)

Given that $y = k^{\alpha} A^{1-\alpha}$, growth in output per worker is given by

$$\frac{\dot{y}}{y} = \alpha \frac{\dot{k}}{k} + (1-\alpha)\frac{\dot{A}}{A} \qquad (18)$$

The evolution of capital is still given by equation $\frac{\dot{k}}{k} = s\frac{y}{k} - \delta - n$ (14)

Capital per worker growth will be constant only if y/k is constant, which will be constant only if $\frac{\dot{y}}{y} = \frac{\dot{k}}{k}$. Using this equality in equation (18) we have

$$\frac{\dot{y}}{y} = \alpha \frac{\dot{y}}{y} + (1-\alpha)g \qquad (19)$$

$$\frac{\dot{y}}{y} = g \qquad (20)$$

or y must grow at a rate g if it is going to grow at a constant rate.

Solow Growth Model

Balanced Growth Path

The steady state of the Solow model will be exactly a situation where growth in y is constant at g. Further, y, k and A will grow at the same rate. The Solow model produces a balanced growth path where everything's growth rate remains constant.

From equation (20) we know that $\frac{\dot{y}}{y} = g$. We also know that $\frac{\dot{y}}{y} = \frac{\dot{k}}{k}$. Therefore, in steady state, equation (14) becomes

$$\frac{\dot{k}}{k} = g = s\frac{y}{k} - \delta - n \qquad (21)$$

Using $y = k^{\alpha}A^{1-\alpha}$ (17), we have

$$\frac{\dot{k}}{k} = g = s\frac{A^{1-\alpha}}{k^{1-\alpha}} - \delta - n \qquad (22)$$

Solving for k we get

$$k = A\left(\frac{s}{\delta+n+g}\right)^{\frac{1}{1-\alpha}} \qquad (23)$$

Solow Growth Model

Steady state with technology

From (23) note that in steady state the ratio $\frac{k}{A} = \left(\frac{s}{\delta+n+g}\right)^{\frac{1}{1-\alpha}}$ is constant because capital and technology grow at the same rate, g.

Output per worker, in steady state, is given by (17) and (23) as

$$y_t = A_t \left(\frac{s}{\delta+n+g}\right)^{\frac{\alpha}{1-\alpha}} \qquad (24)$$

where the time subscripts are explicit to make evident that both variables are growing over time.

Therefore, along a balanced growth path, this economy grows at the rate of technological progress, g. Meanwhile, changes in savings, s, depreciation rate, δ, and population growth, n, will induce temporary changes in growth relative to g as the economy shifts to the new trend line.

Growth empirics: Balanced growth

- Solow model's steady state exhibits **balanced growth** - many variables grow at the same rate.
  - Solow model predicts ***Y***/***L*** and ***K***/***L*** grow at the same rate (***g***), so ***K***/***Y*** should be constant.
  - This is true in the real world.
  - Solow model predicts real wage grows at same rate as ***Y***/***L***, while real rental price is constant.
  - This is also true in the real world.

Correlates of Economic Growth

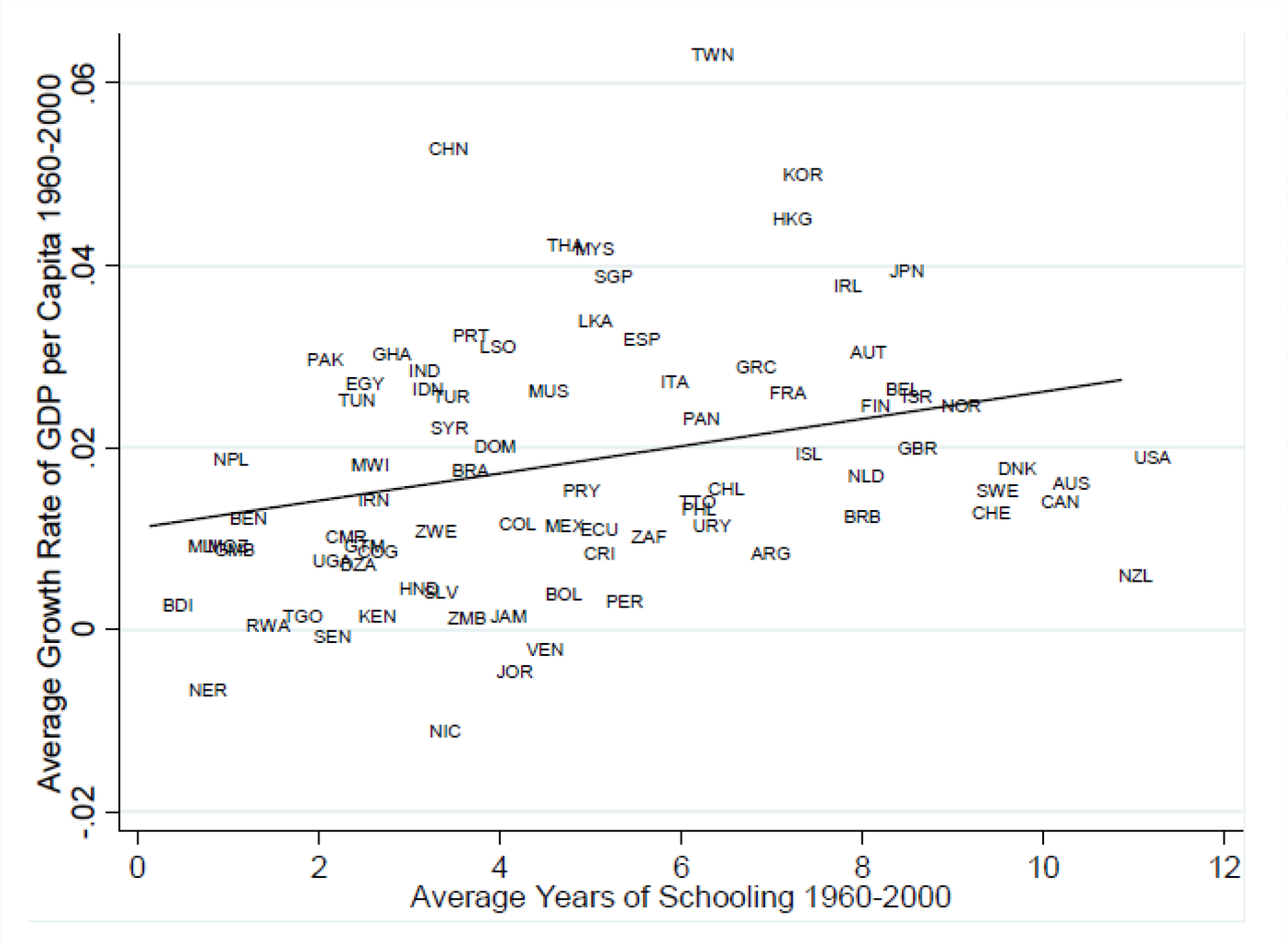


Figure: Schooling and Economic Growth

A Contribution to the Empirics of Economic Growth

N. Gregory Mankiw

David Romer

David N. Weil

The Quarterly Journal of Economics, Vol. 107, No. 2 (May, 1992), pp. 407-437

0. Motivation

➢ Is the Solow growth model consistent with the international variation in the standard of living?

- Is a higher rate of saving associated with a higher income level?

- Is a higher population growth associated with a lower income level?

- Do countries exhibit convergence in standards of living? i.e. do poorer countries tend to grow faster than rich countries?

1. Introduction

➢To a first approximation, predictions of the Solow Growth model are correct:

- A higher rate of saving is associated with a higher income level.
- A higher population growth is associated with a lower income level.
  - 50% of variation in GDP per capita can be explained by these two variables alone.
- Conditional on rate of saving and population growth countries do exhibit (conditional) convergence in standards of living.

➢ While qualitative predictions are confirmed, quantitative effects of savings and population growth are too large to be consistent with textbook Solow growth theory.

MRW Solution

➢ An augmented version of the Solow growth model that allows for human capital accumulation can explain the data however.

- Adding human capital explains why coefficients on savings and population growth are upward biased – human capital is positively correlated with savings.
- When schooling is included in the regression as a measure of human capital MRW find that

  - Coefficients on savings and population growth are now of the correct magnitude for the augmented model

  - The augmented Solow model now explains 80% of the variation in cross-country living standards.

2. The Textbook Solow Model

- Cobb-Douglas production:

$$Y(t) = (K(t))^{\alpha} (A(t)L(t))^{1-\alpha}$$

- Population and technology grow at constant exogenous rates:

$$L(t) = L(0)e^{nt}$$
$$A(t) = A(0)e^{gt}$$

- Constant savings rate:

$$I(t) = S(t) = sY(t)$$

Capital Accumulation

- The evolution of capital per effective unit of labor is governed by:

$$\frac{dk(t)}{dt} \equiv \dot{k}(t) = sy(t) - (n + g + \delta)k(t)$$

- where

$$k(t) = \frac{K(t)}{A(t)L(t)}$$

$$y(t) = \frac{Y(t)}{A(t)L(t)} = (k(t))^{\alpha}$$

- Which implies that

$$\frac{dk(t)}{dt} \equiv \dot{k}(t) = s(k(t))^{\alpha} - (n + g + \delta)k(t)$$

Steady-state

- In steady-state $\frac{dk(t)}{dt} \equiv 0$, which implies

$$k^* = \left(\frac{s}{n+g+\delta}\right)^{\frac{1}{1-\alpha}}$$

- Labor productivity:

$$\frac{Y(t)}{L(t)} = A(t)\left(k^*\right)^{\alpha}$$

- Taking logs we have

$$\log\left(\frac{Y(t)}{L(t)}\right) = \log\left(A(0)\right) + gt + \frac{\alpha}{1-\alpha}\log(s) - \frac{\alpha}{1-\alpha}\log(n+g+\delta)$$

· Model predictions: if $\alpha = \frac{1}{3}$ then $\frac{\alpha}{1\text{-}\alpha} = \frac{1}{2}$, implying that a 10% increase in savings should lead to a 5% increase in GDP per capita.

3. Empirical Specification

- Key question: do data support model's key prediction determining living standards – income per worker is higher in countries with higher savings rates and lower population growth rates.

· Assume that $g + \delta = 0.05$, then countries differ only in terms of savings rate, s, population growth rates, n, and the initial level of technology $A(0)$.

· Let

$$\log(A(0)) = a + \varepsilon$$

where a is common level of technology across countries and ε is country-specific but unobservable (may be explained by institutio nal structure (*cfr*. Hall and Jones 1999), climate, resources endowments, etc.

Empirical Specification

· Key assumption : s and n are uncorrelated with ε.

Assume $t = 0$. This implies the following equation can be estimated

$$\log\left(\frac{Y(0)}{L(0)}\right) = a + \frac{\alpha}{1-\alpha}\log(s) - \frac{\alpha}{1-\alpha}\log(n+g+\delta) + \varepsilon$$

using ordinary least squares.

4. Data and Samples

- Annual data 1960-1988.
- Variables :
  - n = average growth rate of working age population
  - s = average share of real investment in real GDP (including govt. investment).
  - $\frac{Y}{L}$ = real GDP in 1985 / working age population in 1985.
- Sample :
  - All available countries for which oil production is not the dominant industry (98 countries).
  - Drop countries with questionable data (78 countries).
  - OECD (22 countries).

5. Results

TABLE I

ESTIMATION OF THE TEXTBOOK SOLOW MODEL

Dependent variable: log GDP per working-age person in 1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	5.48	5.36	7.97
	(1.59)	(1.55)	(2.48)
ln(I/GDP)	1.42	1.31	0.50
	(0.14)	(0.17)	(0.43)
$\ln(n + g + \delta)$	−1.97	−2.01	−0.76
	(0.56)	(0.53)	(0.84)
$\bar{R}^2$	0.59	0.59	0.01
s.e.e.	0.69	0.61	0.38
Restricted regression:			
CONSTANT	6.87	7.10	8.62
	(0.12)	(0.15)	(0.53)
$\ln(I/GDP) - \ln(n + g + \delta)$	1.48	1.43	0.56
	(0.12)	(0.14)	(0.36)
$\bar{R}^2$	0.59	0.59	0.06
s.e.e.	0.69	0.61	0.37
Test of restriction:			
p-value	0.38	0.26	0.79
Implied α	0.60	0.59	0.36
	(0.02)	(0.02)	(0.15)

Note. Standard errors are in parentheses. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05.

Results Textbook Solow Model

In table I, MRW estimate :

$$\log\left(\frac{Y(0)}{L(0)}\right) = \gamma_0 + \gamma_1 \log(s) + \gamma_2 \log(n + g + \delta) + \varepsilon$$

The model predicts that

$$\gamma_1 = -\gamma_2 = \frac{\alpha}{1-\alpha}$$

MRW find :

- γ_1 is significant and positive.
- γ_2 is significant and negative.
- The data cannot reject that $\gamma_1 = -\gamma_2$.
- The magnitude of $\gamma_1 = 1.42$ implies an estimated $\alpha = \frac{3}{5}$ which is substantially higher than the measured capital share.

6. Adding Human Capital: *H*

- MRW argue that adding human capital to the model may help explain the higher estimated implied elasticity of output with respect to savings.
- To understand this argument, consider a production function of the form:

$$Y(t) = (K(t))^{\alpha}(H(t))^{\beta}(A(t)L(t))^{1-\alpha-\beta}$$

- Assume that both physical and human capital are accumulated according to a constant savings rate:

$$\frac{dk(t)}{dt} = s_k y(t) - (n + g + \delta)k(t)$$

$$\frac{dh(t)}{dt} = s_h y(t) - (n + g + \delta)h(t)$$

Steady-State for Model with Human Capital

- The augmented human capital model will have a balanced growth solution where

$$k^* = \left(\frac{s_k^{1-\beta} s_h^{\beta}}{n+g+\delta}\right)^{\frac{1}{1-\alpha-\beta}}$$

$$h^* = \left(\frac{s_k^{\alpha} s_h^{1-\alpha}}{n+g+\delta}\right)^{\frac{1}{1-\alpha-\beta}}$$

- GDP per capita is then

$$\log\left(\frac{Y(t)}{L(t)}\right) = \log\left(A(0)\right) + gt - \frac{\alpha+\beta}{1-\alpha-\beta}\log\left(n+g+\delta\right)$$
$$+ \frac{\alpha}{1-\alpha-\beta}\log\left(s_k\right) + \frac{\beta}{1-\alpha-\beta}\log\left(s_h\right)$$

7. Empirical Implementation

- If we have data on savings rates for human capital, we could estimate this equation in standard fashion.
- MRW argue that we are more likely to have data on the stock of human capital. If h^* is observable, along the balance growth path we could estimate:

$$\log\left(\frac{Y(t)}{L(t)}\right) = \log(A(0)) + gt + \frac{\alpha}{1-\alpha}\log(s_k) - \frac{\alpha}{1-\alpha}\log(n+g+\delta) + \frac{\beta}{1-\alpha}\log(h^*)$$

- We could then formulate the regression equation

$$\log\left(\frac{Y}{L}\right) = \gamma_0 + \gamma_1 \log(s_k) + \gamma_2 \log(n+g+\delta) + \gamma_3 \log(h^*) + \varepsilon$$

The model predicts that

$$\gamma_1 = -\gamma_2 = \frac{\alpha}{1-\alpha} \quad ; \quad \gamma_3 = \frac{\beta}{1-\alpha}$$

Intuition

- By omitting h^* from our previous regression, we would expect the coefficient γ_1 to be biased. In particular, since human and physical capital are positively correlated, we would expect an upward bias.

- Once we add human capital to the regression, we should obtain a reasonable estimate of γ_1. Given γ_1 we can then infer the implied α and hence from γ_3 we can infer the implied β.

- Human capital is measured as the percentage of the population with a secondary education.

8. Results Solow Model with Human Capital

TABLE II

ESTIMATION OF THE AUGMENTED SOLOW MODEL

Dependent variable: log GDP per working-age person in 1985

Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	6.89	7.81	8.63
	(1.17)	(1.19)	(2.19)
ln(I/GDP)	0.69	0.70	0.28
	(0.13)	(0.15)	(0.39)
$\ln(n + g + \delta)$	−1.73	−1.50	−1.07
	(0.41)	(0.40)	(0.75)
ln(SCHOOL)	0.66	0.73	0.76
	(0.07)	(0.10)	(0.29)
$\bar{R}^2$	0.78	0.77	0.24
s.e.e.	0.51	0.45	0.33
Restricted regression:			
CONSTANT	7.86	7.97	8.71
	(0.14)	(0.15)	(0.47)
$\ln(I/GDP) - \ln(n + g + \delta)$	0.73	0.71	0.29
	(0.12)	(0.14)	(0.33)
$\ln(SCHOOL) - \ln(n + g + \delta)$	0.67	0.74	0.76
	(0.07)	(0.09)	(0.28)
$\bar{R}^2$	0.78	0.77	0.28
s.e.e.	0.51	0.45	0.32
Test of restriction:			
p-value	0.41	0.89	0.97
Implied α	0.31	0.29	0.14
	(0.04)	(0.05)	(0.15)
Implied β	0.28	0.30	0.37
	(0.03)	(0.04)	(0.12)

Note. Standard errors are in parentheses. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05. SCHOOL is the average percentage of the working-age population in secondary school for the period 1960–1985.

Summary of the Results with Human Capital

- Adding human capital to the regression increases the R^2 from 0.6 to 0.8, i.e. it adds significan t explanatory power implying that ommited variable bias is a problem.
- Once we add the human capital, we obtain an estimate of γ_1 that is much lower and an implied estimate of α that is much closer to $\frac{1}{3}$.
- The estimates also imply that $\beta = \frac{1}{3}$.
- So, the total capital share is $\alpha + \beta = \frac{2}{3}$.
- With human capital in the model, the Solow growth framework fits the data in terms of explaining cross-country variation in GDP per capita as a function of savings in physical capital, population growth and schooling.

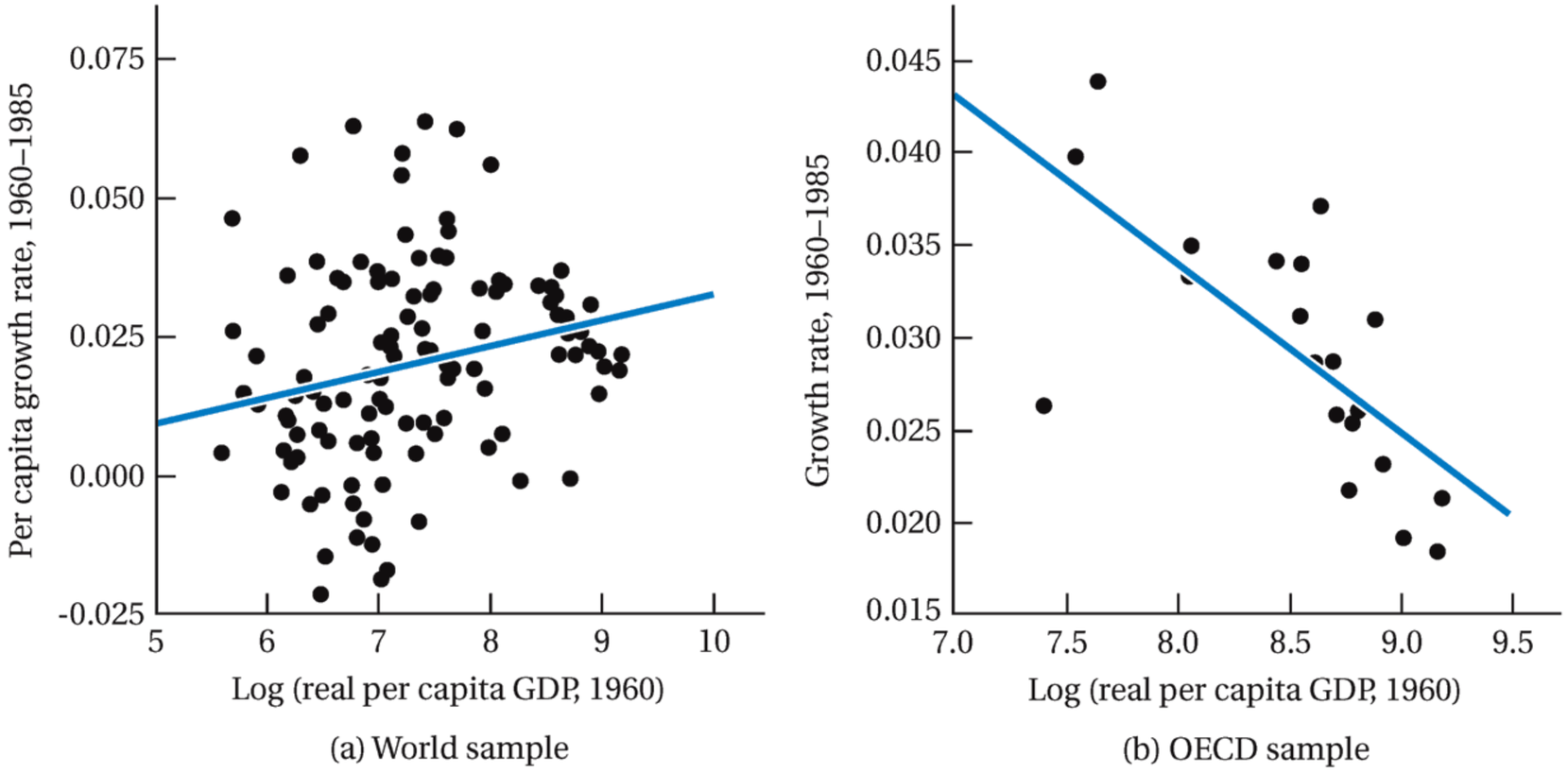
9. Convergence

- Solow model predicts that, other things equal, “poor” countries (with lower ***Y***/***L*** and ***K***/***L***) should grow faster than “rich” ones.

- If true, then the income gap between rich & poor countries would shrink over time, causing living standards to “converge.”

- In real world, many poor countries do NOT grow faster than rich ones. Does this mean the Solow model fails?

Convergence among OECD Countries but Divergence in the World as a Whole



Source: Robert Barro and Xavier Sala-i-Martin, *Economic Growth* (New York: McGraw-Hill, 1995), p. 27. Reprinted with permission.

Growth empirics: Convergence

- Solow model predicts that, other things equal, "poor" countries (with lower ***Y***/***L*** and ***K***/***L***) should grow faster than "rich" ones.

- No, because "other things" aren't equal.
  - In samples of countries with similar savings & pop. growth rates, income gaps shrink about 2% per year.
  - In larger samples, after controlling for differences in saving, pop. growth, and human capital, incomes converge by about 2% per year.

9.1 Results Unconditional Convergence

TABLE III

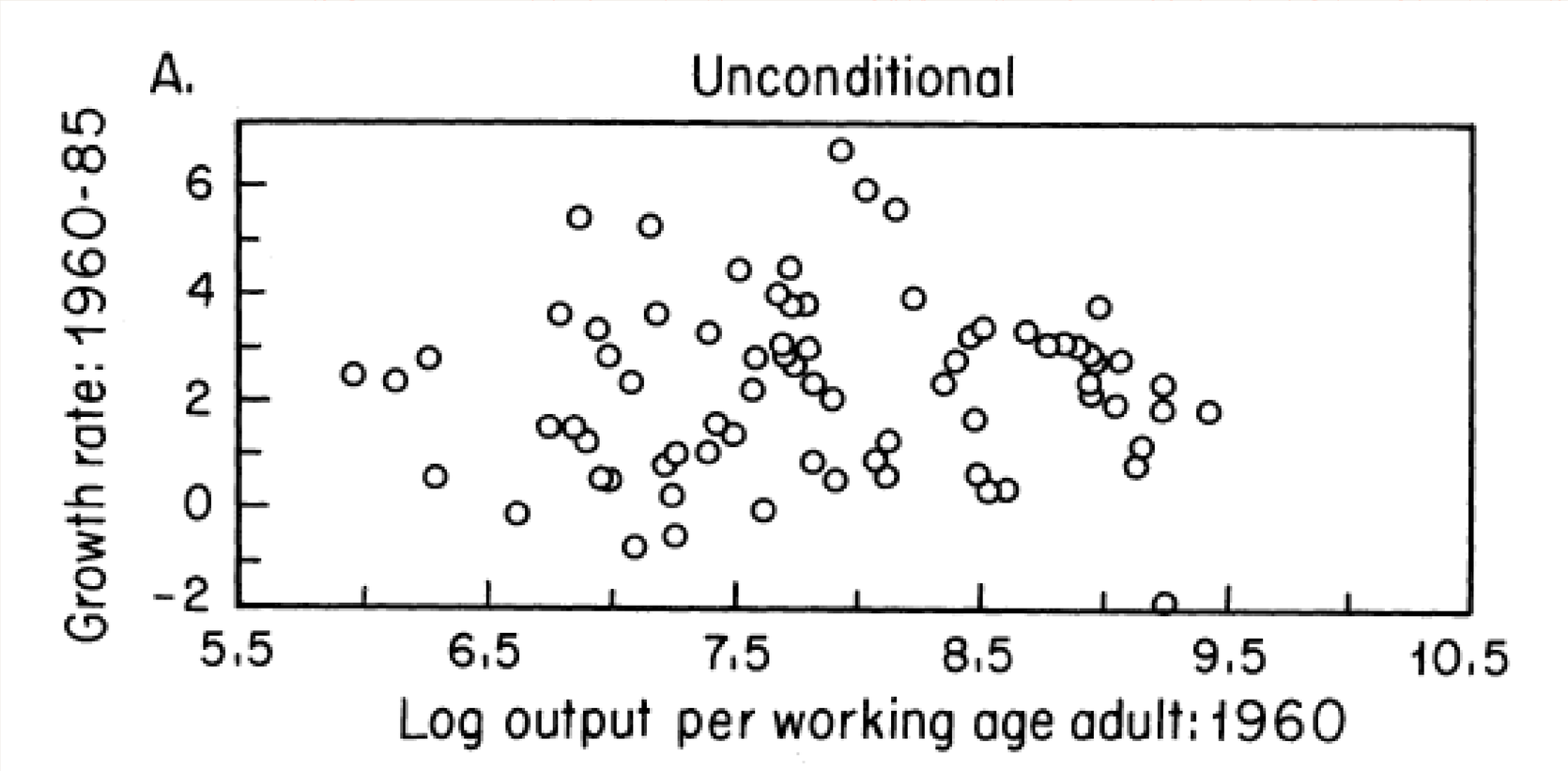
TESTS FOR UNCONDITIONAL CONVERGENCE

Dependent variable: log difference GDP per working-age person 1960–1985

Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	−0.266	0.587	3.69
	(0.380)	(0.433)	(0.68)
ln(Y60)	0.0943	−0.00423	−0.341
	(0.0496)	(0.05484)	(0.079)
$\overline{R}^2$	0.03	−0.01	0.46
s.e.e.	0.44	0.41	0.18
Implied λ	−0.00360	0.00017	0.0167
	(0.00219)	(0.00218)	(0.0023)

Note. Standard errors are in parentheses. Y60 is GDP per working-age person in 1960.

Results Unconditional Convergence



9.2 Results Conditional Convergence

TABLE IV

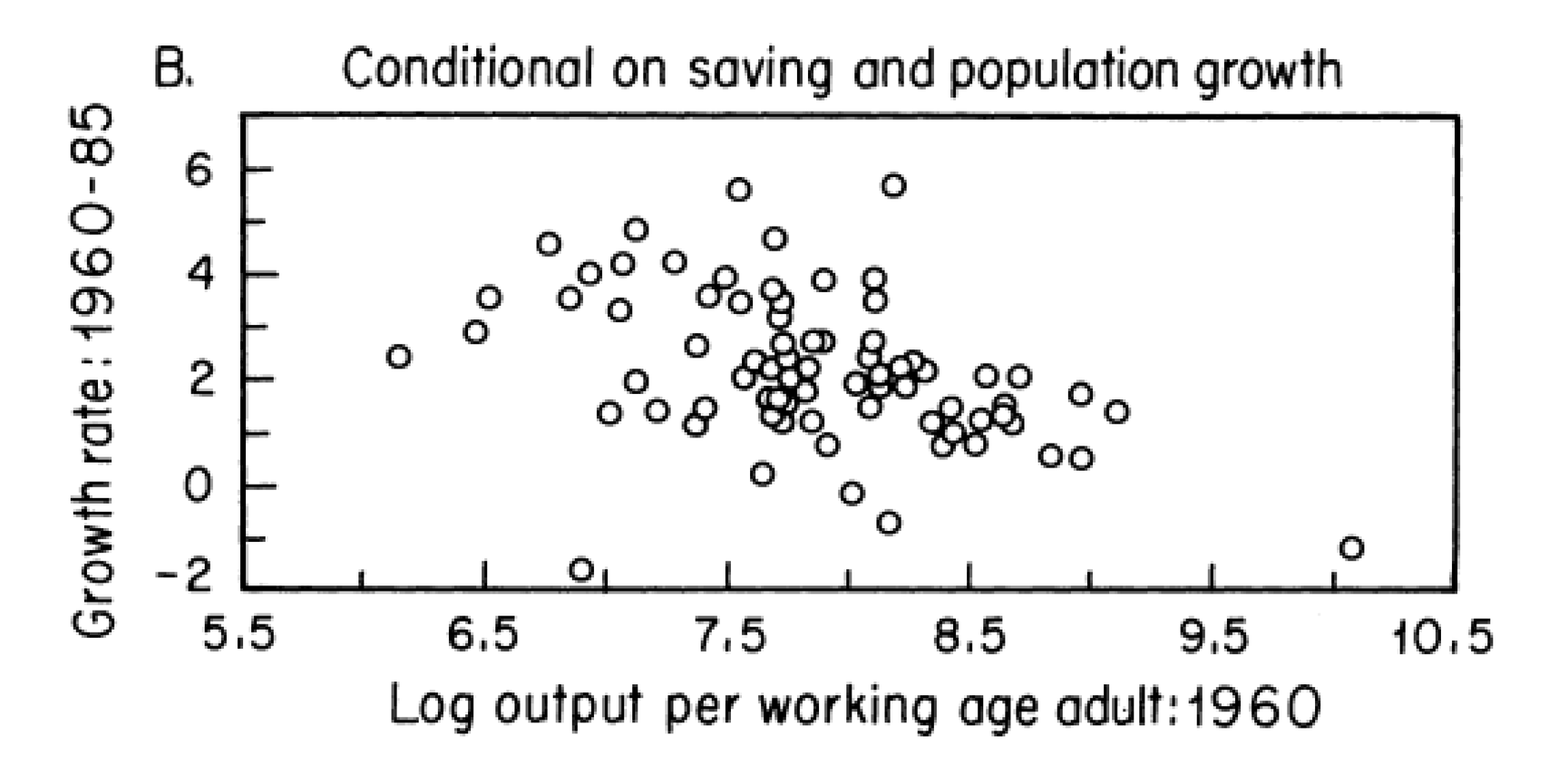
TESTS FOR CONDITIONAL CONVERGENCE

Dependent variable: log difference GDP per working-age person 1960–1985

Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	1.93	2.23	2.19
	(0.83)	(0.86)	(1.17)
ln(Y60)	−0.141	−0.228	−0.351
	(0.052)	(0.057)	(0.066)
ln(I/GDP)	0.647	0.644	0.392
	(0.087)	(0.104)	(0.176)
$\ln(n + g + \delta)$	−0.299	−0.464	−0.753
	(0.304)	(0.307)	(0.341)
$\bar{R}^2$	0.38	0.35	0.62
s.e.e.	0.35	0.33	0.15
Implied λ	0.00606	0.0104	0.0173
	(0.00182)	(0.0019)	(0.0019)

Note. Standard errors are in parentheses. Y60 is GDP per working-age person in 1960. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05.

Results Conditional Convergence



9.3. Results Conditional Convergence adding Schooling

TABLE V

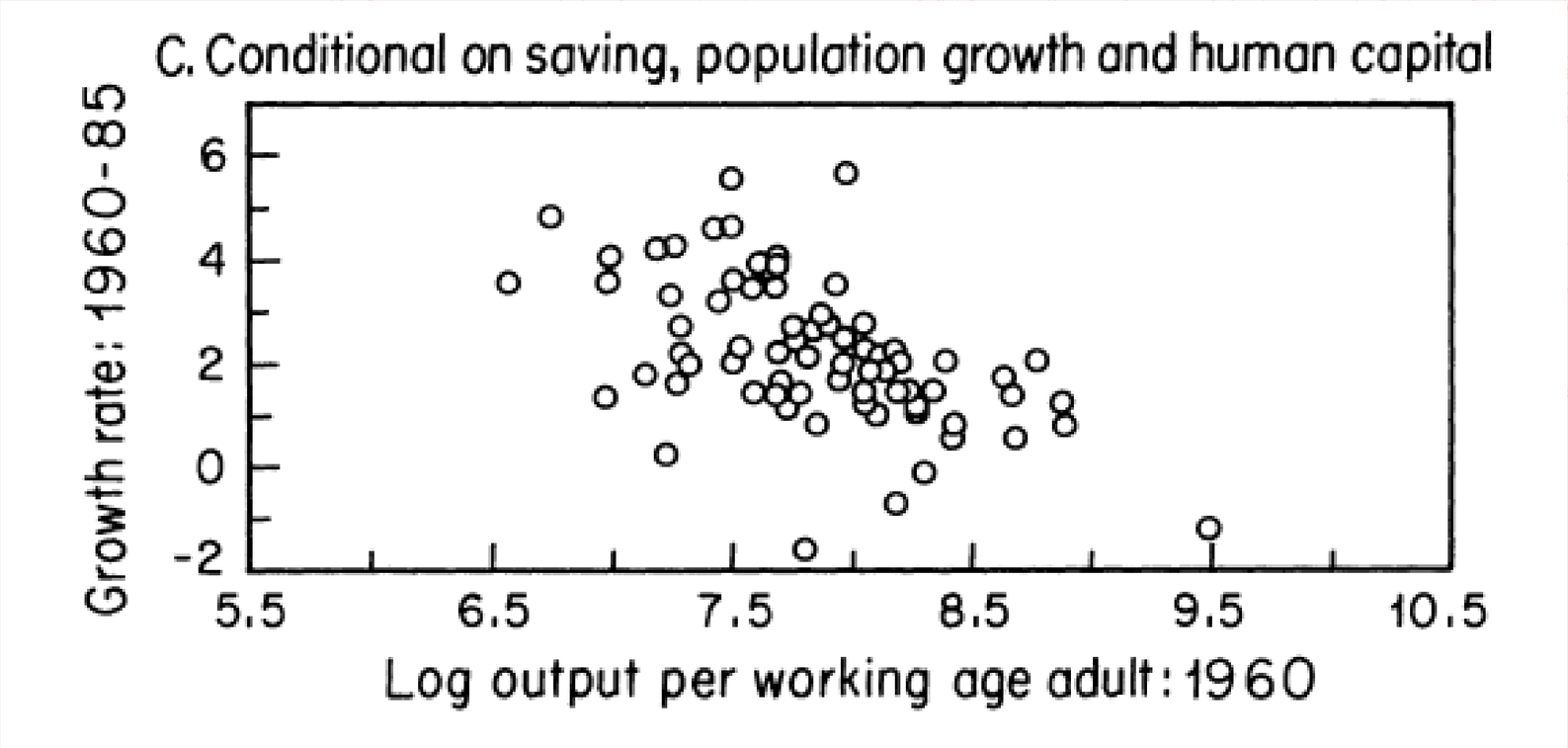
TESTS FOR CONDITIONAL CONVERGENCE

Dependent variable: log difference GDP per working-age person 1960–1985

Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	3.04	3.69	2.81
	(0.83)	(0.91)	(1.19)
ln(Y60)	−0.289	−0.366	−0.398
	(0.062)	(0.067)	(0.070)
ln(I/GDP)	0.524	0.538	0.335
	(0.087)	(0.102)	(0.174)
$\ln(n + g + \delta)$	−0.505	−0.551	−0.844
	(0.288)	(0.288)	(0.334)
ln(SCHOOL)	0.233	0.271	0.223
	(0.060)	(0.081)	(0.144)
$\bar{R}^2$	0.46	0.43	0.65
s.e.e.	0.33	0.30	0.15
Implied λ	0.0137	0.0182	0.0203
	(0.0019)	(0.0020)	(0.0020)

Note. Standard errors are in parentheses. Y60 is GDP per working-age person in 1960. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05. SCHOOL is the average percentage of the working-age population in secondary school for the period 1960–1985.

Results Conditional Convergence adding Schooling



Technological Growth

The Quarterly Journal of Economics, Vol. 114, No. 1 (February, 1999), pp. 83-116

Technology Drives Growth and Economics of Ideas

- In the Solow model, the only thing that produces trend growth is technology, A. We said that A grew at some rate g over time. Where does g come from?
- A could be considered as ideas of how to combine inputs more efficiently. Better ideas imply higher output per worker, se we need to continue to have new ideas to grow.
- Economics of ideas are different from economics of goods and services: ideas are non-rival and ideas are (generally) non-exclusive.
- As ideas are so different, the economics of producing ideas are different from out standard world of perfect competition.
- Ideas are non-rival. It needs to be produced once and then can be used by anyone. Ideas have high fixed costs and low (zero) marginal costs → ideas have increasing returns to scale → average cost of the idea > marginal cost of reproducing the idea ∴ ideas will only be produced if someone can charge more than marginal cost → there must be imperfect competition.

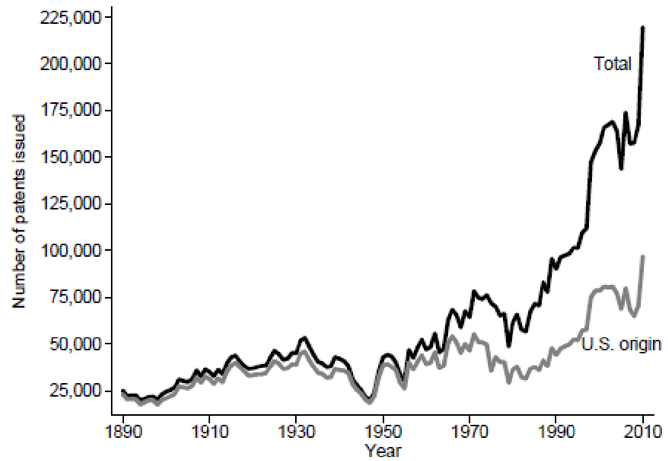
Imperfect Competition

- Firms that produce ideas, or goods that embody ideas earn profits by charging a price for the good over marginal cost.
- The profits are there to make up for the large fixed cost to coming up with the idea.
- Can only sustain these profits by preventing others from using the idea – patents, branding, marketing, copyrights, etc.
- Without protection from the idea, firms will not ear profits. Without profits, firms will not undertake fixed cost of R&D to invent new product.
- So the growth of ideas, A, depends on there being imperfect competition. Economic growth fundamentally relies on market imperfections.

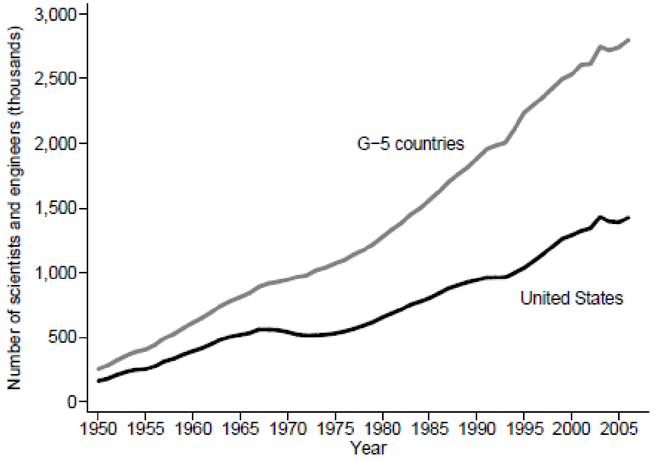
Population and Ideas

- Population is a positive for ideas
- More people means more researchers, thinkers, inventors, etc.
- Ideas are non-rival. So more people dies not mean fewer ideas for each of us. We can all use all the ideas all the time. This is different from rival factors of production as capital.
- More people means larger markets. Larger markets mean more profits for firms that use ideas. More profits means more investment in R&D.
- All together, population (or scale) will be a positive contributor to growth.

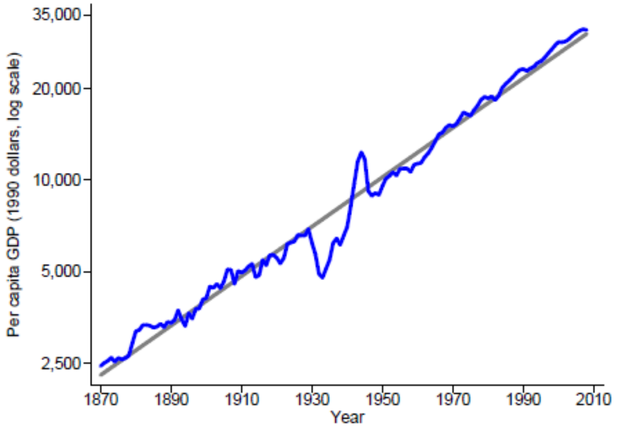
Number of patents issued in the U.S.



Number of scientists and engineers doing R&D in U.S. and G-5.



Note that while the number of ideas, and number of researchers is climbing very rapidly, trend growth in the U.S. has remained very steady. Will want to ensure our model captures that.



Modelling Ideas

- We want some model of the accumulation of ideas. A is the index of "ideas". How do we accumulate new ideas? The model will have two parts:
  1) Mechanics of accumulating technology – which will depend on R&D effort exerted.
  2) Economics of how much effort to exert on R&D.
- Start with the mechanics – to determine the long-run growth rate g in the economy.
- Assume that the stock of technology, A,evolves according to

$$\dot{A} = \bar{\theta} L_A \qquad (1)$$

where L_A are the number of workers (researchers) doing R&D on new technologies, and $\bar{\theta}$ is the rate at which each researcher finds a new idea.

$\bar{\theta}$ depends on the total number of researchers working and the existing level of technology, A, according to

$$\bar{\theta} = \theta L_A^{\lambda-1} A^{\phi} \qquad (2)$$

so that

$$\dot{A} = \theta L_A^{\lambda} A^{\phi} \qquad (3)$$

or the growth rate of A is

$$\frac{\dot{A}}{A} = \theta \frac{L_A^{\lambda}}{A^{1-\phi}} \qquad (4)$$

- $0 < \lambda < 1$ captures how research responds to the number of researchers, $\lambda > 0$ implies adding researchers raises technology growth, but $\lambda < 1$ implies some congestion/duplication.
- $\phi < 1$ captures effect of technology level on change in technology. $\phi > 0$ → "standing on shoulders", $\phi < 0$ → fishing out.

Long-run Growth Rate

- What is the growth rate of A along a balanced growth path? Meaning, what is the growth rate of A such that $\frac{\dot{A}}{A}$ remains constant?
- From $\frac{\dot{A}}{A} = \theta \frac{L_A^{\lambda}}{A^{1-\phi}}$ (4), $\frac{\dot{A}}{A}$ will be constant if L_A^{λ} grows at exactly the same rate as $A^{1-\phi}$. Take logs and derivatives of both sides

$$ln\left(\frac{\dot{A}}{A}\right) = ln\theta + \lambda L_A - (1-\phi)lnA \qquad (5)$$

$$\frac{\partial ln\left(\frac{\dot{A}}{A}\right)}{\partial t} = \lambda \frac{\dot{L_A}}{L_A} - (1-\phi)\frac{\dot{A}}{A} \qquad (6)$$

- Along the Balanced Growth Path, for the growth rate of A to be constant, the left-hand side of (6) should be zero, or

$$\frac{\dot{A}}{A} = \frac{\lambda}{1-\phi}\frac{\dot{L_A}}{L_A} \qquad (7)$$

- Assume that the growth rate of researchers is proportional to the growth rate of population such that $\frac{\dot{L_A}}{L_A} = n$, which implies that in the long-run the trend growth rate is given by

$$\frac{\dot{A}}{A} = \frac{\lambda}{1-\phi}n \qquad (8)$$

- The trend growth is a positive function of n as the faster the population growth, the faster technology grows, as more people means more researchers and more researchers means more ideas.

A special case based on Romer (1994)

- Paul Romer (1994), "The Origins of Endogenous Growth," Journal of Economic Perspectives, 8 (1): 3-22.

- Romer's original growth model takes $\frac{\dot{A}}{A} = \theta \frac{L_A^{\lambda}}{A^{1-\phi}}$ (4) and set $\lambda = 1$ and $\phi = 1$, for

$$\dot{A} = \theta L_A A \qquad (9)$$

or

$$\frac{\dot{A}}{A} = \theta L_A \qquad (10)$$

- For Romer, an increase in the number of researchers led to an increase in the growth rate of technology. If this were true, then the trend growth rate of output per worker should have been rising over the 20th century.

- Because growth rate is so steady over modern history, the assumption $\phi = 1$ cannot be right. Almost certain that $\phi < 1$.

The production of ideas and comparative statics

- So far, this description has been completely mechanical. g depends on the values of λ, ϕ, and n. But there are also economic choices to be made. We will focus on the fraction of the total labor force used to do research, $0 < s_R < 1$, where

$$L_A = s_R L \qquad (10)$$

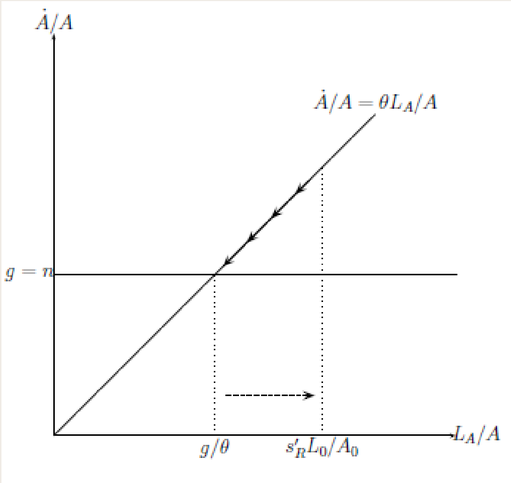
- If $\phi < 1$, then s_R has no effect on the long-run growth rate of A, and only on the level of A.
- From $\frac{\dot{A}}{A} = \theta \frac{L_A^{\lambda}}{A^{1-\phi}}$ (4), substitute (10) and assume $\lambda = 1$ and $\phi = 0$, so

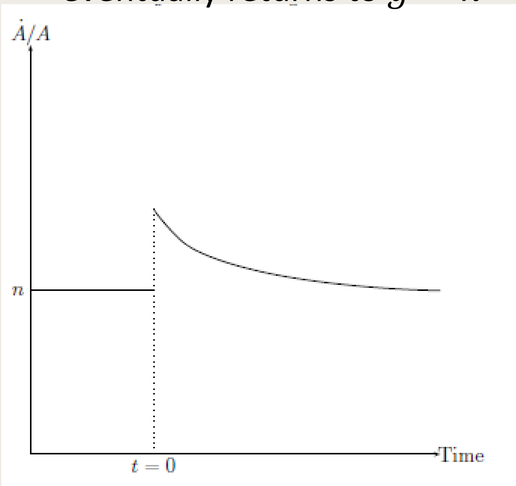
$$\frac{\dot{A}}{A} = \frac{\theta s_R L}{A} \qquad (11)$$

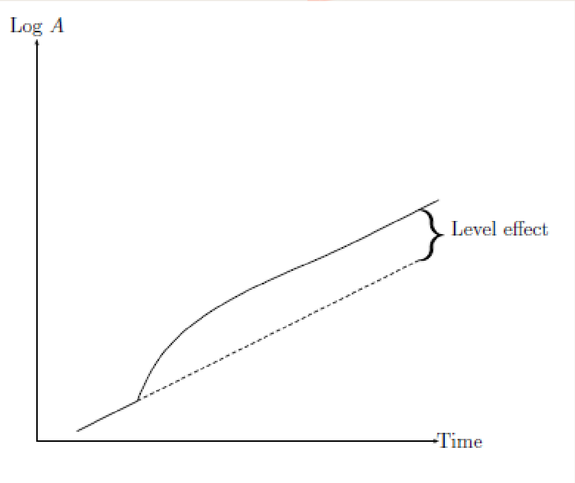
- What happens if s_R goes up to s_R'?

This temporary shift in s_R has a temporary effect on the growth rate of A, but does not change the long-run growth rate that eventually returns to $g = n$

The shift up in s_R does permanently raise the level of A.







Embedding Technology in the Solow Model

We now have a model for technology growth. How does that fit in the Solow model? One change to make. With s_R people researching, only $(1 - s_R)$ are working in the production of final goods, so

$$y = k^{\alpha}(A(1 - s_R)L)^{1-\alpha} \tag{12}$$

and along the balanced growth path

$$y(t) = \dot{A}(t)(1 - s_R)\left(\frac{s}{\delta+n+g}\right)^{\frac{\alpha}{1-\alpha}} \tag{13}$$

Fewer people working means lower output per worker.

Along the balanced growth path, $\frac{\dot{A}}{A} = g = \frac{s_R L}{A}$, therefore from (13) we have

$$y(t) = \frac{s_R L(t)}{g}(1 - s_R)\left(\frac{s}{\delta+n+g}\right)^{\frac{\alpha}{1-\alpha}} \tag{14}$$

s_R has conflicting effects on the level of output per capita. You can do so much research that the level of $y(t)$ actually goes down because of a lack of workers.

Conflicting effects of population. There are scale effects, so if $L(t)$ is bigger, we have higher output per worker. More people means more ideas.

Faster population growth means a lower level of output per worker. Recall that $g = n$. Fast-growing population increase A so fast that it makes it harder to find new technologies. Also dilutes capital per worker as in Solow.

Endogenous Growth: The Romer Model

Paul Romer (1994), "The Origins of Endogenous Growth," Journal of Economic Perspectives, 8 (1): 3-22.

Economics of Technology

- Solow model with $\frac{\dot{A}}{A} = \theta \frac{(s_R L)^{\lambda}}{A^{1-\phi}}$ (4') to describe growth of A gives us a way of describing trend growth as a function of population growth.
- But what determines s_R? What determines how much effort the economy puts into R&D? Romer model will provide answers. Has three components.
  1. Final goods sector – competitive
  2. Intermediate goods sector – imperfect competition
  3. Research sector – comes up with new ideas for intermediate goods.

1. Final Goods Firms

Final goods are produced by perfectly competitive firms using this function

$$Y = L_Y^{1-\alpha} \int_0^A x_j^\alpha dj \qquad (1)$$

where L_Y is the number of workers in the final goods sector, A indexes the number of intermediate goods used, x_j is the amount of intermediate good j used by the final goods sector.

The final goods firms try to maximize profits

$$\max_{L_Y x_j} L_Y^{1-\alpha} \int_0^A x_j^\alpha dj - wL_Y - \int_0^A p_j x_j dj \qquad (2)$$

which gives the first-order conditions

$$w = (1-\alpha)\frac{Y}{L_Y} \qquad (3)$$

$$p_j = \alpha L_Y^{1-\alpha} x_j^{\alpha-1} \qquad \forall\ j \qquad (4)$$

2. Intermediate Goods Firms

The intermediate goods come from separate firms. The intermediate goods firms are monopolists – only one firm can produce each intermediate good.

Intermediate good firms produce those goods by transforming capital into them one-for-one. One unit of capital produces one unit of intermediate goods.

The profits of any given intermediate good firm j are

$$\pi_j = p_j(x_j)x_j - rx_j \tag{5}$$

where $p_j(x_j)$ is the demand function of the final goods firm for the intermediate good j.

Because the firm is a monopolist, they consider how demand changes as they produce more x_j. The first-order condition is

$$p'(x)x + p(x) = r \tag{6}$$

which can be rearranged as

$$p = \frac{1}{1+\frac{p\prime(x)x}{p}}r \tag{7}$$

where $\frac{p\prime(x)x}{p}$ is the elasticity of demand. From the demand function of intermediate goods (4), the elasticity is $\alpha - 1$. Thus the firm charges a mark-up over marginal cost

$$p = \frac{1}{1+\alpha-1}r = \frac{1}{\alpha}r \tag{8}$$

The intermediate good firms earn profits that give the reason for innovation. The profit-maximization problem is identical for all intermediate good firms, they charge the same price and given identical demand function they sell the same amount.

Aggregate Output

Putting this all together. The total amount of intermediate goods produced must be equal to the capital stock, as each intermediate good firm uses capital to produce, so

$$K = \int_0^A x_j dj \tag{9}$$

and because all $x_j = x$, it must be that

$$x = \frac{K}{A} \tag{10}$$

Final output is

$$Y = L_Y^{1-\alpha} \int_0^A x_j^\alpha dj = L_Y^{1-\alpha} A x^\alpha \tag{11}$$

or using (10)

$$Y = K^\alpha (AL_Y)^{1-\alpha} \tag{12}$$

Aggregate final output is described just like in the Solow model, with a different underlying market structure.

Distribution of Income and Profits

From the final goods firms first-order condition (3) we know that

$$wL_Y = (1-\alpha)Y \tag{13}$$

which means that

$$\int_0^A p_j x_j dj = \alpha Y \tag{14}$$

From (8) $p_j = \frac{r}{\alpha}$ for all intermediate goods firms, and from (10) $x_j = \frac{K}{A}$. Plugging these two conditions in (14) we get that

$$\int_0^A \frac{r}{\alpha}\frac{K}{A} dj = \alpha Y \tag{15}$$

which solves down to

$$rK = \alpha^2 Y \tag{16}$$

Summing individual profits (5), total profits of all intermediate good firms are

$$\int_0^A \pi_j \, dj = \int_0^A p_j x_j dj - \int_0^A r x_j dj = \alpha Y - \alpha^2 Y = \alpha(1-\alpha)Y \tag{17}$$

For any individual intermediate good firm, since they are identical, profits are a fraction $\frac{1}{A}$ of total profits, and provide the incentives to innovation

$$\pi_j = \frac{\alpha(1-\alpha)Y}{A} \tag{18}$$

Total output is distributed across three types of payments, a fraction to capital (16), a fraction to labor (13), and a fraction to profits (17), according to

$$rK + wL_Y + \int_0^A \pi_j \, dj = \alpha^2 Y + (1-\alpha)Y + \alpha(1-\alpha)Y = Y \tag{19}$$

3. Research Sector

Individuals can work at trying to invent a new type of intermediate good.

For each researcher, they take the chance of discovering a new idea as given, and equal to $\bar{\theta} = \theta L_A^{\lambda-1} A^{\phi}$, that researchers take as given.

If a researcher discovers a new idea, they receive a patent on that idea. They own the rights to produce that new intermediate good.

The researcher can then sell that patent to a firm that wants to be the monopolist producing that intermediate good (or the researcher could start their own firm).

We assume that people can move bank and forth from research to working in the final goods sector, so the return to research (the expected value of getting a patent) will be equal to the return to working (the wage).

We use an arbitrage equation to get the value of a patent, P_A

$$rP_A = \pi + \dot{P}_A \qquad (20)$$

where rP_A is the amount earned from investing P_A dollars in the bank. This is the alternative to buying the patent. π is the profits earned from owing the patent, solved for before. $\dot{P}_A$ is the change in the value of the patent if you own it – you could resell it later for a profit (or loss).

Balanced Growth Path for Patents

Re-arranging the arbitrage equation (20) we get

$$r = \frac{\pi}{P_A} + \frac{\dot{P}_A}{P_A} \tag{21}$$

Along the balanced growth path, we know that r is constant. Therefore, the ratio $\frac{\pi}{P_A}$ must be constant, and $\frac{\dot{P}_A}{P_A}$ must be constant.

Along the balanced growth path, we know that

$$\pi = \alpha(1-\alpha)\frac{Y}{A} = \alpha(1-\alpha)\frac{Y}{L}\frac{L}{A} \tag{22}$$

Along the balanced growth path, $\frac{Y}{L}$ grows at the rate g. L grows at the rate n. So π must grow at the rate n.

This implies that $\frac{\dot{P}_A}{P_A} = n$, to keep $\frac{\pi}{P_A}$ constant. Putting that into the arbitrage equation gives

$$r = \frac{\pi}{P_A} + n \tag{23}$$

or

$$P_A = \frac{\pi}{r-n} \tag{24}$$

which says that the value of a patent is the present discounted value of the profits from owing the patent.

Labor Market Equilibrium

The last thing to do is solve for the fraction of workers who do research. People can work or do research. Doing research earns

$$w_R = \bar{\theta} P_A \tag{25}$$

or the chance of making a discovery times the value of a discovery.

Working earns the wage paid in the final goods sector

$$w_Y = (1-\alpha)\frac{Y}{L_Y} \tag{26}$$

Equilibrium requires that both wages equalize

$$w_R = w_Y \tag{27}$$

or

$$\bar{\theta} P_A = (1-\alpha)\frac{Y}{L_Y} \tag{28}$$

or using (24)

$$\bar{\theta}\frac{\pi}{r-n} = (1-\alpha)\frac{Y}{L_Y} \tag{29}$$

and (22)

$$\bar{\theta}\frac{\alpha(1-\alpha)\frac{Y}{A}}{r-n} = (1-\alpha)\frac{Y}{L_Y} \tag{30}$$

which simplifies to

$$\frac{\bar{\theta}}{A}\frac{\alpha}{r-n} = \frac{1}{L_Y} \tag{31}$$

Equilibrium s_R

Therefore, from $\frac{\bar{\theta}}{A}\frac{\alpha}{r-n} = \frac{1}{L_Y}$ (31) and using $\dot{A} = \theta L_A A$ (9) from previous section and that along the balanced growth path $\frac{\dot{A}}{A} = g$, such that $\frac{\bar{\theta}}{A} = \frac{g}{L_A}$ we have

$$\frac{g}{L_A}\frac{\alpha}{r-n} = \frac{1}{L_Y} \tag{32}$$

$$\frac{\alpha g}{r-n} = \frac{L_A}{L_Y} \tag{33}$$

$$\frac{\alpha g}{r-n} = \frac{s_R}{1-s_R} \tag{34}$$

which can be solved for the share of people doing research, s_R

$$s_R = \frac{1}{1+\frac{r-n}{\alpha g}} \tag{35}$$

where the discount rate $r - n$ has a negative effect on s_R, as the higher the discount rate, lower the present discounted value of patents. Meanwhile, the growth rate g as a positive effect on s_R as if people are finding new ideas quickly doing research is more lucrative.

Optimal s_R?

Is the share s_R optimal in the sense that it maximizes output per workers along the balanced growth path? In general no. Recall that

$$y(t) = \frac{s_R L(t)}{g}(1 - s_R)\left(\frac{s}{\delta+n+g}\right)^{\frac{\alpha}{1-\alpha}} \qquad \text{(36 = 14 previous)}$$

along the balanced growth path. There are conflicting effects of s_R, and there is some value of s_R that gives us the highest level of $y(t)$.

Why does the market NOT necessarily get us the optimal value of s_R?

Value of patents does not capture the effect of a new idea on the future flow of ideas. It may either lower (if $\phi < 0$) or raise (if $\phi > 0$) the productivity of other researchers by inventing a new idea (raising A). If $\phi > 0$, the market under-provides research.

Value of patents does not capture the duplication of effort. With $\lambda > 0$ the research of a person, slow downs other persons research, and the market over-provides research.

Imperfect competition. The incentive to innovate is based on profits only, not the value of the new intermediate good to producing final goods. So the market tends to under-provide research.

Some estimates that R&D has a social, or total, return of 40% - 60%, exceeding any private return on investment in R&D from added profits.

Creative Destruction: A Schumpeterian model

Ricardo Caballero (2008), "Creative Destruction," in The New Palgrave Dictionary of Economics.

Creative Destruction

- Creative destruction refers to the incessant product and process innovation mechanism by which new production units replace outdated ones.
- This restructuring process permeates major aspects of macroeconomic performance, not only long-run growth but also economic fluctuations, structural adjustment and the functioning of factor markets.
- Over the long run, the process of creative destruction accounts for over 50 per cent of productivity growth. At business cycle frequency, restructuring typically declines during recessions, and this add a significant cost to downturns.
- Obstacles to the process of creative destruction can have severe short- and long-run macroeconomic consequences.

Innovation Through Improving Existing Products

The Romer model thought of innovation as adding new intermediate goods to the market, but once invented, each good was fixed in quality.

The Schumpeterian model conceives innovation as improving existing products with newer versions replacing old versions (iPhone 14 vs iPhone 13 vs iPhone 6). Hence the “creative destruction”.

Schumpeterian growth changes the mechanical details of growth, but not the general conclusions

1) The long-run trend growth rate depends on population growth.

2) The allocation of workers to research may not be optimal.

One advantage of the Schumpeterian model is that it explicitly allows to think about firm dynamics, or the creation and destruction of firms over time.

Mechanics of Growth

Final goods are produced using

$$Y = K^{\alpha}(A_i L_Y)^{1-\alpha} \qquad (1)$$

where A_i is the productivity of the latest version of technology. So $A_2 > A_1$ and $A_{100} > A_{99}$.

A_t moves up in discrete jumps, so

$$A_{i+1} = (1+\gamma)A_i \qquad (2)$$

where γ is the "step size", or how much productivity rises each time we innovate.

Growth occurs when we innovate, but that doesn't always happen. The growth rate of A_i from innovation to innovation is

$$\frac{A_{i+1}-A_i}{A_i} = \gamma \qquad (3)$$

but this is not how fast A_t grows over time.

The chance that any given researcher will produce an innovation at any given moment is

$$\bar{\mu} = \theta \frac{L_A^{\lambda-1}}{A_i^{1-\phi}} \qquad (4)$$

or the probability of innovation depends on the same forces as before: duplication due to other researchers and the spillovers of A_i on research.

For the economy as a whole, the probability of making an innovation depends on how many researchers are working, so

$$P(Innovation) = \bar{\mu} L_A = \theta \frac{L_A^{\lambda}}{A_i^{1-\phi}} \qquad (5)$$

Long-run Growth Rate

The growth rate of this economy is not immediately obvious.

- At any given point in time, no one may have innovated, so growth is zero.
- When someone does innovate, A_i jumps by γ, so growth is very rapid in that moment.
- Look at average growth over long period, smoothing this out
- The expected growth rate along the balanced growth path (smoothed line) is

$$E\left[\frac{\dot{A}}{A}\right] = \gamma\bar{\mu}L_A = \gamma\theta\frac{L_A^{\lambda}}{A_i^{1-\phi}} \qquad (6)$$

where γ tells us how much A jumps when an innovation occurs and $\bar{\mu}L_A$ tells us the expected value of the number of jumps.

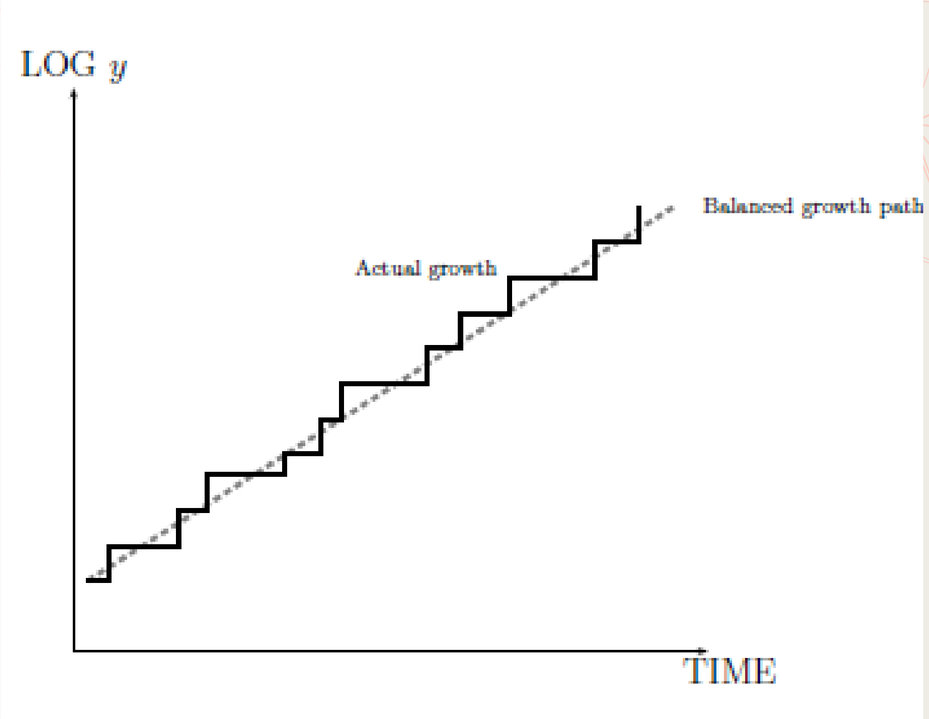
Using this, what is the expected growth rate of A along the BGP?

$$0 = \lambda\frac{\dot{L}_A}{L_A} - (1-\phi)E\left[\frac{\dot{A}}{A}\right] \qquad (7)$$

which gives $\frac{\dot{L}_A}{L_A} = n$ along the BGP, means that

$$E\left[\frac{\dot{A}}{A}\right] = g = \frac{\lambda}{1-\phi}n \qquad (8)$$

which is identical to what we had in the Romer model.



Final Goods Production

Final goods produced using

$$Y = L_Y^{1-\alpha} A_i^{1-\alpha} x_i^{\alpha} \qquad (9)$$

where x_i is a single intermediate (or capital) good used in the final goods sector. It is indexed by i because each intermediate good has a specific productivity level, A_i associated with it.

Final good firms will maximize profits by choosing which version of x_i to use and how many units.

$$\max_{L_Y, x_i} L_Y^{1-\alpha} A_i^{1-\alpha}\, x_i^{\alpha} - wL_Y - p_i x_i \qquad (10)$$

The first-order necessary conditions are:

$$w = (1-\alpha)\frac{Y}{L_Y} \qquad (11)$$

$$p_i = \alpha L_Y^{1-\alpha} A_i^{1-\alpha} x_i^{\alpha-1} \qquad (12)$$

As in the Romer model, the elasticity of demand for the intermediate good is $\alpha - 1$.

Intermediate Good Firms

Intermediate good firms are monopolists at producing their version of the intermediate good. As before, they transform one unit of capital into one unit of the intermediate good. Their profits are

$$\pi_i = p_i(x_i)x_i - rx_i \qquad (13)$$

The first-order condition is

$$p_i'(x_i)x_i + p_i(x_i) = r \qquad (14)$$

which can be rearranged as

$$p_i = \frac{1}{1+\frac{p_i'(x_i)x_i}{p_i}} r \qquad (15)$$

which given the elasticity we found for final goods firm demand gives

$$p_i = \frac{1}{\alpha} r \qquad (16)$$

Similar to the Romer model, intermediate good firms charge a markup over marginal costs. This generates the profits that will motivate innovation. The price they charge does not depend on the version of the intermediate good they produce. So, all versions of x_i sell for the same price, final good firms only buy the best one (highest A_i).

This set-up ensures the creative destruction in the economy. Once a new innovation occurs (a new x_{i+1}) with a higher productivity, the old intermediate good (x_i) firm goes out of business and the new firm takes over completely.

Aggregate Output

Given that only one intermediate good firm operates at a time, it must be that

$$K = x_i \tag{17}$$

meaning that aggregate output is

$$Y = K^{\alpha}(A_i L_Y)^{1-\alpha} \tag{18}$$

which is the same as the standard production function.

$$wL = (1-\alpha)Y \tag{19}$$

$$rK = \alpha^2 Y \tag{20}$$

$$\pi_i = \alpha(1-\alpha)Y \tag{21}$$

with the only difference being that all profits accrue to the one operating intermediate good firm.

Research Sector

Again, we want to understand the incentive to do research. Inventing a new version x_{i+1} gives you a patent on that good you can sell (or use to be the monopolist).

We use an arbitrage equation to get the value of a patent, P_A

$$rP_A = \pi + \dot{P}_A - \bar{\mu} L_A P_A \tag{22}$$

where rP_A is again the amount earned from investing P_A dollars in the bank. $\pi + \dot{P}_A$ is the value of the patent: profits plus capital gains. $\bar{\mu} L_A P_A$ captures the fact that with probability $\bar{\mu} L_A$, you will be replaced as the monopolist by the next innovator, so it is a negative.

Rearranging we get

$$r = \frac{\pi}{P_A} + \frac{\dot{P}_A}{P_A} - \bar{\mu} L_A \tag{23}$$

and for convenience let $\mu = \bar{\mu} L_A$ so

$$r = \frac{\pi}{P_A} + \frac{\dot{P}_A}{P_A} - \mu \tag{24}$$

Patents Along BGP

As in the Romer model, we want to consider the value of a patent along the BGP, where r is constant.

This implies that π and P_A must grow at the same rate.

We know that profits are $\pi = \alpha(1-\alpha)Y$, so profits grow at the rate $g + n$.

We know that $g = \gamma\bar{\mu}L_A = \gamma\mu$ along the BGP

so we have that

$$r = \frac{\pi}{P_A} + \gamma\mu + n - \mu \tag{25}$$

which solves to

$$P_A = \frac{\pi}{r - n + \mu(1-\gamma)} \tag{26}$$

Again, patents are the present discounted value of profits. Now the discount rate is higher because of μ, which captures the chance of being replaced.

Note that no existing monopolist would ever buy the new patent. Why? Because they have to sacrifice their existing profits, meaning they will not pay as much for the new patent. So always new firms coming into existence. “Arrow Replacement Effect”.

Labor Market Equilibrium

Again, individuals can either do research to get the next idea or work in the final goods sector. They move back and forth until the returns to these two activities are identical, or

$$(1-\alpha)\frac{Y}{L_Y} = \bar{\mu}P_A \qquad (27)$$

where $\bar{\mu}$ is the chance that an individual will innovate, and P_A is the value of that innovation to them. Using (26) to replace P_A we get

$$(1-\alpha)\frac{Y}{L_Y} = \bar{\mu}\frac{\pi}{r-n+\mu(1-\gamma)} \qquad (28)$$

Using $\pi_i = \alpha(1-\alpha)Y$ (21) we get

$$(1-\alpha)\frac{Y}{L_Y} = \bar{\mu}\frac{\alpha(1-\alpha)Y}{r-n+\mu(1-\gamma)} \qquad (29)$$

$$\frac{1}{L_Y} = \bar{\mu}\frac{\alpha}{r-n+\mu(1-\gamma)} \qquad (30)$$

Using $\bar{\mu} = \frac{\mu}{L_A}$

$$\frac{1}{L_Y} = \frac{\mu}{L_A}\frac{\alpha}{r-n+\mu(1-\gamma)} \qquad (31)$$

$$\frac{L_A}{L_Y} = \mu\frac{\alpha}{r-n+\mu(1-\gamma)} \qquad (32)$$

$$\frac{s_R}{1-s_R} = \mu\frac{\alpha}{r-n+\mu(1-\gamma)} \qquad (33)$$

$$s_R = \frac{1}{1+\frac{r-n+\mu(1-\gamma)}{\alpha\mu}} \qquad (34)$$

Equilibrium s_R

In the Romer model we had $s_R = \frac{1}{1+\frac{r-n}{\alpha g}}$ (35 Romer), now we have

$$s_R = \frac{1}{1+\frac{r-n+\mu(1-\gamma)}{\alpha\mu}} \qquad (34)$$

Therefore, we have the same discount factor $r - n$ that has a negative effect on s_R, as the higher the discount rate, lower the present discounted value of patents.

μ has two effects, the first one $\mu(1-\gamma)$ captures the fact that as the probability of innovation goes up, the value of patents declines due to replacement effects; the second effect $\alpha\mu$ captures the fact that as the probability of innovation goes up, you are more likely to get a patent in the first place. On net, the second effect "wins". You get a patent now, and will only be replaced later, so if μ goes up, s_R goes up.

Comparing to the Romer model, the long-run growth rate is identical at $g = \frac{\lambda n}{1-\phi}$. The difference in the level of income along the BGP implied by the value of s_R.

The Schumpeterian model has a higher s_R if $g < r - n$. In this case the discount rate is very large and so I care most about profits in the immediate future and little about the fact that I might be replaced some day. So more people do research than in Romer.

The Schumpeterian model has lower s_R if $g > r - n$. In this case the discount rate is low, so people do care about the future replacement a lot. Hence s_R would be low compared to Romer.

Higher s_R is not necessarily optimal. There is no sense in which Romer or Schumpeter is "better", they are different ways of conceiving of the growth process.

Why Do Some Countries Produce So Much More Output Per Worker Than Others?

Robert E. Hall

Charles I. Jones

The Quarterly Journal of Economics, Vol. 114, No. 1 (February, 1999), pp. 83-116

I. Introduction

➢ Explain and quantify the factors that cause the large observed cross-country differences in output worker.

- Are the differences due to: … ?
  - Physical capital *(K)*
  - Human capital *(H)*
  - Technology *(A)*

- Example: US has 35 times more output per worker than Nigeria.
  - Physical capital intensity accounts for a factor of 1.5;
  - Human capital intensity accounts for a factor of 3.1;
  - The remaining factor of 7.7 is due to the Solow residual, i.e., (1.5)(3.1)(7.7) = 35.

Main Questions and Premise

- Why do some countries invest more than others in physical and human capital?

- Why are some countries so much more productive than others?

- **Major Premise:** differences in capital accumulation, productivity, and therefore output per worker are fundamentally related to differences in social infrastructure across countries.

Social Infrastructure

- Good social infrastructure encourages productive activities.
  -Capital accumulation, skill acquisition, invention and technology transfer.
  -Private return to such activity is close to social return which is high.

- Good social infrastructure works by protecting against diversion;
  - e.g. Thievery, squatting, mafia protection.

- While governments are necessary to encourage productive activity they may also be the major source of diversion:
  - Public diversion occurs through expropriation, corruption, confiscatory taxes.
  - Regulation and laws may protect against diversion but may also be the main vehicle of diversion.

Methodology

- Focus in differences in levels of GDP per worker.

- Key idea – in long-run, technology transfer implies countries will tend to have the same growth rate but do not converge in levels.

- Empirically, evidence suggests that differences in growth rates are transitory – there is evidence for conditional convergence.

Causation

- Focus on the role of social infrastructure in promoting capital accumulation and productivity:

$$\text{Social Infrastructure} \Rightarrow \text{Inputs, Productivity} \Rightarrow \text{Output per worker}$$

- There may be feedback – poor countries have low social infrastructure because they cannot afford to build the right institutions.

- To control for such feedback, use instrumental variables approach:
  - Geography and linguistic characteristics of a country are fixed (no feedback) but predict social infrastructure.
  - Thus they are good "instruments" for social infrastructure.

- Intuition: geography and linguistic characteristics are measures of closeness to Western Europe, the first region in the world to implement broadly a social infrastructure favorable to production.

II. Levels Accounting

- Decompose differences in output per worker to differences in inputs and differences in productivity using a Cobb-Douglas accounting framework:

$$Y_i = K_i^{\alpha}\left(A_i H_i\right)^{1-\alpha}$$

where

$$Y_i = \text{Output}$$

$$K_i = \text{Physical capital}$$

$$H_i = \text{Human capital augmented labor}$$

$$A_i = \text{Labor-augmenting technology}$$

Human Capital and Schooling

- Human capital determined by

$$H_i = e^{\phi(E_i)} L_i$$

where

$$L_i = \text{Units of labor}$$

$$E_i = \text{Years of schooling}$$

The function $\phi(E_i)$ measures efficiency of schooling. $\phi(0) = 0,$

while $\phi'(E_i) > 0$ measures the marginal productivity (return) to schooling.

Decomposition

- Production function:

$$Y = K^{\alpha}(AH)^{1-\alpha}$$

- Divide by Y^{α}

$$Y^{1-\alpha} = \left(\frac{K}{Y}\right)^{\alpha}(AH)^{1-\alpha}$$

- Take exponents and then divide by L

$$\frac{Y}{L} = \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} A\frac{H}{L}$$

- The level of output per worker can be decomposed into capital-output ratio, technology and human capital.

- Intuition: along balanced growth path, K/Y is a constant that primarily reflects differences in savings across countries.

Measuring Physical Capital

- Construct capital stock for each country using a perpetual inventory method:

$$K_{t+1} = I_t + (1-\delta)K_t$$

- Starting with an initial value

$$K_0 = \frac{I_0}{g+\delta}$$

- Effect of initial value is small if *t* is large.

Measuring Human Capital

- Measure ϕ*(E)* using summary estimates from Mincer wage regressions:

$$\phi'(E) = 0.134 \qquad 0 < E \leq 4$$
$$\phi'(E) = 0.101 \qquad 4 < E \leq 8$$
$$\phi'(E) = 0.068 \qquad 8 < E$$

- For example, if a country has an average of 4 years of schooling then:

$$\phi(4) = 0.134 * 4 = 0.536$$

and human capital is

$$H = e^{\phi(4)} = e^{0.536} = 1.71$$

- If a country has an average of 12 years of schooling then

$$\phi(12) = 0.134 * 4 + 0.101 * 4 + 0.068 * 4 = 1.21$$

and human capital is

$$H = e^{\phi(12)} = e^{1.21} = 3.36$$

Data Decomposition

- Measure output per worker from Penn World Tables.

- Given measures of *K/Y* and *H* we can then do the decomposition

$$\frac{Y_i}{L_i} = \left(\frac{K_i}{Y_i}\right)^{\frac{\alpha}{1-\alpha}} A_i \frac{H_i}{L_i}$$

where we choose α=1/3

- Each factor is normalized relative to the United States value, i.e. H_i measures country *i*'s human capital relative to US human capital.

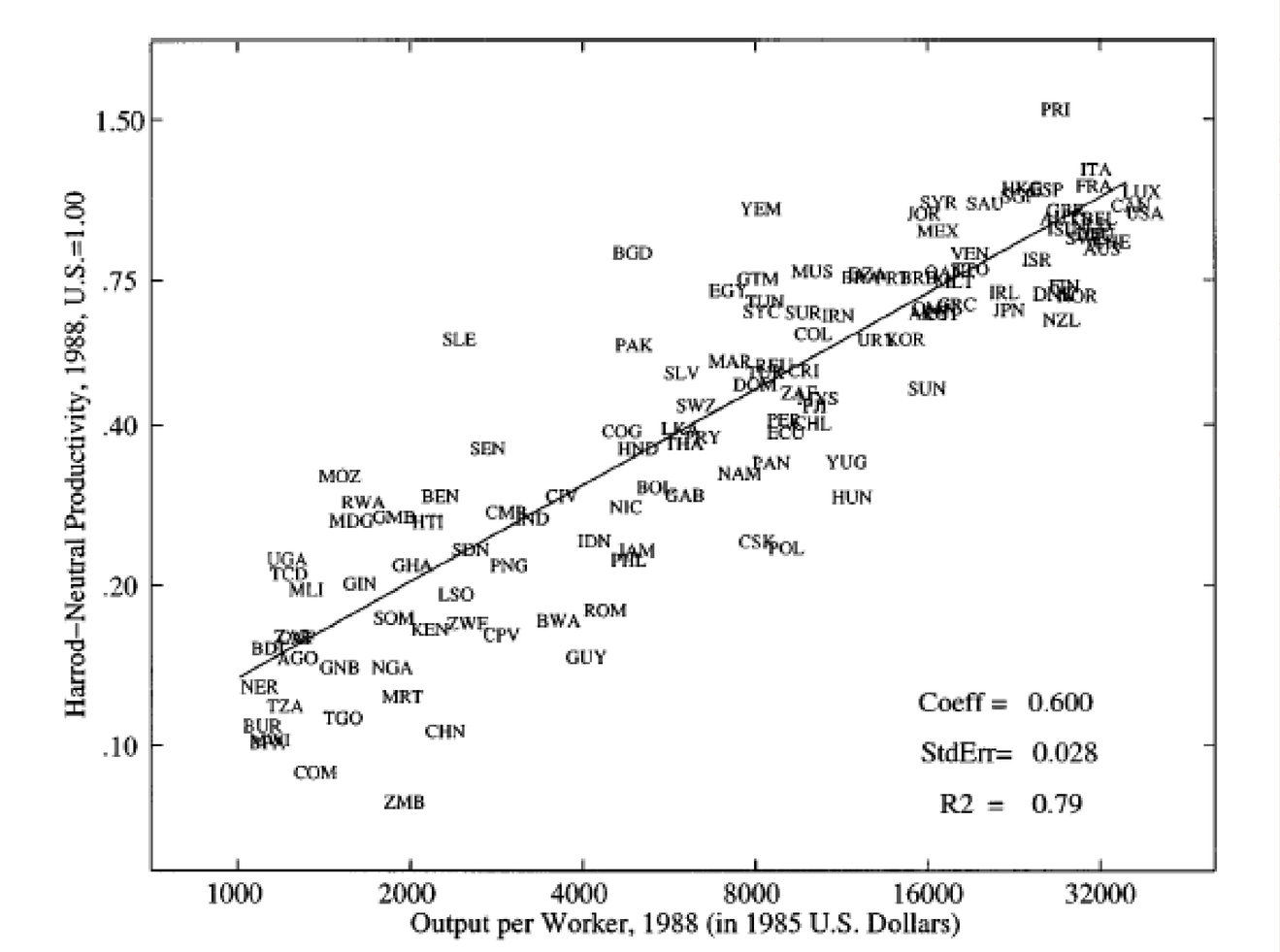
Data Decomposition

TABLE I
PRODUCTIVITY CALCULATIONS: RATIOS TO U. S. VALUES

Country	Y/L	Contribution from $(K/Y)^{\alpha/(1-\alpha)}$	H/L	A
United States	1.000	1.000	1.000	1.000
Canada	0.941	1.002	0.908	1.034
Italy	0.834	1.063	0.650	1.207
West Germany	0.818	1.118	0.802	0.912
France	0.818	1.091	0.666	1.126
United Kingdom	0.727	0.891	0.808	1.011
Hong Kong	0.608	0.741	0.735	1.115
Singapore	0.606	1.031	0.545	1.078
Japan	0.587	1.119	0.797	0.658
Mexico	0.433	0.868	0.538	0.926
Argentina	0.418	0.953	0.676	0.648
U.S.S.R.	0.417	1.231	0.724	0.468
India	0.086	0.709	0.454	0.267
China	0.060	0.891	0.632	0.106
Kenya	0.056	0.747	0.457	0.165
Zaire	0.033	0.499	0.408	0.160
Average, 127 countries:	0.296	0.853	0.565	0.516
Standard deviation:	0.268	0.234	0.168	0.325
Correlation with Y/L (logs)	1.000	0.624	0.798	0.889
Correlation with A (logs)	0.889	0.248	0.522	1.000

The elements of this table are the empirical counterparts to the components of equation (3), all measured as ratios to the U. S. values. That is, the first column of data is the product of the other three columns.

Productivity and Output per Worker



Differences in productivity are very similar to differences in output per worker.

Results from Data Decomposition

- Five richest countries have output per worker that is 32 times five poorest countries.

- Most of the difference in output per worker is due to difference in productivity *A*.

- Capital-output ratios have a modest effect in explaining cross-country differences in output per worker.

  - Investment rates across countries differ by a factor of 3.

  - Since $\alpha/(1-\alpha)=0.5$ this can account for a factor $(3)^{0.5}=1.73$ difference in output per worker.

Results from Data Decomposition

- Human capital differences also have a modest effect in explaining output per worker differences:

  - On average five richest countries have 8 years more schooling than five poorest.

  - This difference would account for a factor of approximately 2 in output per worker (*e.g.* $H(12)/H(4) = 1.96$).

- So in summary, physical and human capital each account for approximately a factor of 2 in differences in output per worker between richest and poorest countries.

- Productivity accounts for a factor of 8.

Comparison with Mankiw, Romer and Weil

- MRW estimate the production function using econometric methods.

  - To do so, they must assume that the level of technology is uncorrelated with savings rates in physical and human capital.

- H&J choose a parameter for the production function and then perform data decomposition.

  - Their results imply that savings rates and the stock of human capital are highly correlated with the level of technology.

Social Infrastructure and Output per Worker

Posit a relationship of the form:

$$y = \alpha + \beta S + \varepsilon$$

where $y = \log\left(\frac{Y}{L}\right)$ and S is a measure of social infrastructure.

We wish to measure β and determine the degree to which social infrastructure determines cross-country differences in $\frac{Y}{L}$.

Endogeneity:

$$S = \gamma + \delta y + \theta X + \eta$$

Social infrastructure is determined by the endogenous factor $\frac{Y}{L}$ and the exogenous factor X.

Key Identifying Assumption

Assume that:

$$E(X\varepsilon) = 0$$

This assumption implies that X predicts the social infrastructure but has no direct effect on output per worker (through its correlation with the unmeasured residual ε).

We also assume $E(\varepsilon) = 0$ without loss of generality.

Instrumental Variables

This is known as an instrumental variables procedure.

To estimate

$$y = \alpha + \beta S + \varepsilon$$

X is a valid instrument for S if :

- Instrument relevance : X and S are correlated (X predicts S)
- Instrument exogeneity : X and ε are uncorrelated (X does not have an independent effect on y).

Measuring Social Infrastructure (*S*)

- Government Anti Diversion Policy Index (GADP).
  - law and order
  - bureaucratic quality
  - corruption
  - risk of expropriation
  - government repudiation of debt

- Index of openness (Sachs-Warner) measures percent of time a country is "open" to trade. Open to trade defined as
  - nontariff barriers cover less than 40% of trade
  - average tariff rates are less than 40%
  - black market premiums are less than 20%
  - not socialist
  - govt. does not monopolize major exports

Instrument Choices (*X*)

- Instruments:
  - percent of population that speaks a Western European language (English, French, Spanish, Portuguese, German)
  - distance from the Equator

○ Both measures are designed to capture influence of Western Europe on social infrastructure – Western Europe more likely to expand to remote areas with land (farther from Equator) and/or to climates similar to Western Europe.

○ Also the amount of trade that one would predict based on distance between countries and population as an additional instrument.

Estimation Results

TABLE II
BASIC RESULTS FOR OUTPUT PER WORKER
$\log Y/L = \alpha + \beta\tilde{S} + \tilde{\epsilon}$

Specification	Social infrastructure	OverID test *p*-value test result	Coeff test *p*-value test result	$\hat{\sigma}_{\tilde{\epsilon}}$
1. Main specification	5.1432	.256	.812	.840
	(.508)	Accept	Accept	
Alternative specifications to check robustness				
2. Instruments:	4.998	.208	.155	.821
Distance, Frankel-Romer	(.567)	Accept	Accept	
3. No imputed data	5.323	.243	.905	.889
79 countries	(.607)	Accept	Accept	
4. OLS	3.289	—	.002	.700
	(.212)		Reject	

The coefficient on Social infrastructure reflects the change in log output per worker associated with a one-unit increase in measured social infrastructure. For example, the coefficient of 5.14 means than a difference of .01 in our measure of social infrastructure is associated with a 5.14 percent difference in output per worker. Standard errors are computed using a bootstrap method, as described in the text. The main specification uses distance from the equator, the Frankel-Romer instrument, the fraction of the population speaking English at birth, and the fraction of the population speaking a Western European language at birth as instruments. The OverID test column reports the result of testing the overidentifying restrictions, and the Coeff test reports the result of testing for the equality of the coefficients on the *GADP* policy index variable and the openness variable. The standard deviation of log Y/L is 1.078.

Instrumental variables estimation implies a 1% increase in social infrastructure leads to a 5% increase in *y* (3.5% with OLS).

Additional Results

TABLE III
REDUCED-FORM REGRESSIONS

Regressors	Dependent variables: Social infrastructure	Dependent variables: Log (output per worker)
Distance from the equator, (0,1) scale	0.708	3.668
	(.110)	(.337)
Log of Frankel-Romer predicted trade share	0.058	0.185
	(.031)	(.081)
Fraction of population speaking English	0.118	0.190
	(.076)	(.298)
Fraction of population speaking a European language	0.130	0.995
	(.050)	(.181)
R^2	.41	.60

N = 127. Standard errors are computed using a bootstrap method, as described in the text. A constant term is included but not reported.

All instruments of social infrastructure help predict output per worker.

Additional Results

TABLE IV
RESULTS FOR log K/Y, log H/L, and log A
$Component = \alpha + \beta\tilde{S} + \tilde{\epsilon}$

	Dependent variable		
	$\frac{\alpha}{1-\alpha}$ log K/Y	log H/L	log A
Social infrastructure	1.052	1.343	2.746
	(.164)	(.171)	(.336)
OverID test (p)	.784	.034	.151
Test result	Accept	Reject	Accept
$\hat{\sigma}_{\tilde{\epsilon}}$	.310	.243	.596
$\hat{\sigma}_{\text{Depvar}}$	.320	.290	.727

Estimation is carried out as in the main specification in Table II. Standard errors are computed using a bootstrap method, as described in the text.

Social infrastructure is an important determinant of each of the components of labor productivity.

Additional Results

TABLE VI
ROBUSTNESS RESULTS
$\log Y/L = \alpha + \beta\tilde{S} + \lambda \textit{ Added Variable} + \tilde{\epsilon}$

Specification	Social infrastructure	Additional variable	OverID test p-value test result	$\hat{\sigma}_{\tilde{\epsilon}}$
1. $\tilde{S} = GADP$	5.410	. . .	.006	.769
	(.394)		Reject	
2. $\tilde{S}$ = years open	4.442	. . .	.131	1.126
	(.871)		Accept	
3. Distance from equator	5.079	0.062	.129	.835
	(2.61)	(2.062)	Accept	
4. Ethnolinguistic fractionalization	5.006	−0.223	.212	.816
(N = 113)	(.745)	(.386)	Accept	
5. Religious affiliation (N = 121)	4.980	See	.478	.771
	(.670)	Note	Accept	
6. Log (population)	5.173	0.047	.412	.845
	(.513)	(.060)	Accept	
7. Log (C-H density)	5.195	−0.546	.272	.850
	(.539)	(1.11)	Accept	
8. Capitalist system indicator	6.354	−1.057	.828	.899
variable	(1.14)	(.432)	Accept	
9. Instruments: main set plus	4.929	. . .	.026	.812
continent dummies	(.388)		Reject	

See notes to Table II. Instruments are the same as in Table II, except where noted. Additional variables are discussed in the text. The coefficients on the religious variables in line 5, followed by standard errors, are Catholic (0.992,.354), Muslim (0.877,.412), Protestant (0.150,.431), and Hindu (0.839,1.48).

Results are robust to including distance to equator as separate explanatory variable for output per worker –it has only a small effect independent of its effect on social infrastructure.

How much variation in *y* can social infrastructure explain?

TABLE V
FACTORS OF VARIATION: MAXIMUM/MINIMUM

	Y/L	$(K/Y)^{\alpha/(1-\alpha)}$	H/L	A
Observed factor of variation	35.1	4.5	3.1	19.9
Ratio, 5 richest to 5 poorest countries	31.7	1.8	2.2	8.3
Predicted variation, only measurement error	38.4	2.1	2.6	7.0
Predicted variation, assuming $r^2_{\tilde{S},S} = .5$	25.2	1.9	2.3	5.6

The first two rows report actual factors of variation in the data, first for the separate components and then for the geometric average of the five richest and five poorest countries (sorted according to Y/L). The last two rows report predicted factors of variation based on the estimated range of variation of true social infrastructure. Specifically, these last two rows report $\exp(r\beta_{IV}(\tilde{S}_{\max} - \tilde{S}_{\min}))$, first with $r = .800$ and second with $r^2 = .5$.

- Problem here is that social infrastructure is measured with error so we don't see the true variation.
  - one can compare OLS to IV estimates to infer something about degree of measurement error.
- If there is no measurement error in social infrastructure, estimates imply difference if factor of 95 in terms of labor productivity due to social infrastructure.
- If there is measurement error of plausible magnitude, we get differences on the order of 25-35 –this is what we see in the data.

The New Kaldor Facts: Ideas, Institutions, Population, and Human capital

Charles I. Jones

Paul M. Romer

American Economic Journal: Macroeconomics, Vol. 2, No. 1 (January 2010), pp. 224-245

www.bankandfinance.net

Nicholas Kaldor Economic Growth Facts

- Stated in 1961 by Nicholas Kaldor to summarize what economists had learned from their analysis of 20th century growth and to frame the research agenda going forward:

1. Labor productivity has grown at a sustained rate.
2. Capital per worker has also grown at a sustained rate.
3. The real interest rate, or return on capital, has been stable.
4. The ratio of capital to output has also been stable.
5. Capital and labor have captured stable shares of national income.
6. Among the fast-growing countries of the world, there is an appreciable variation in the rate of growth “of the order of 2- 5 percent.”

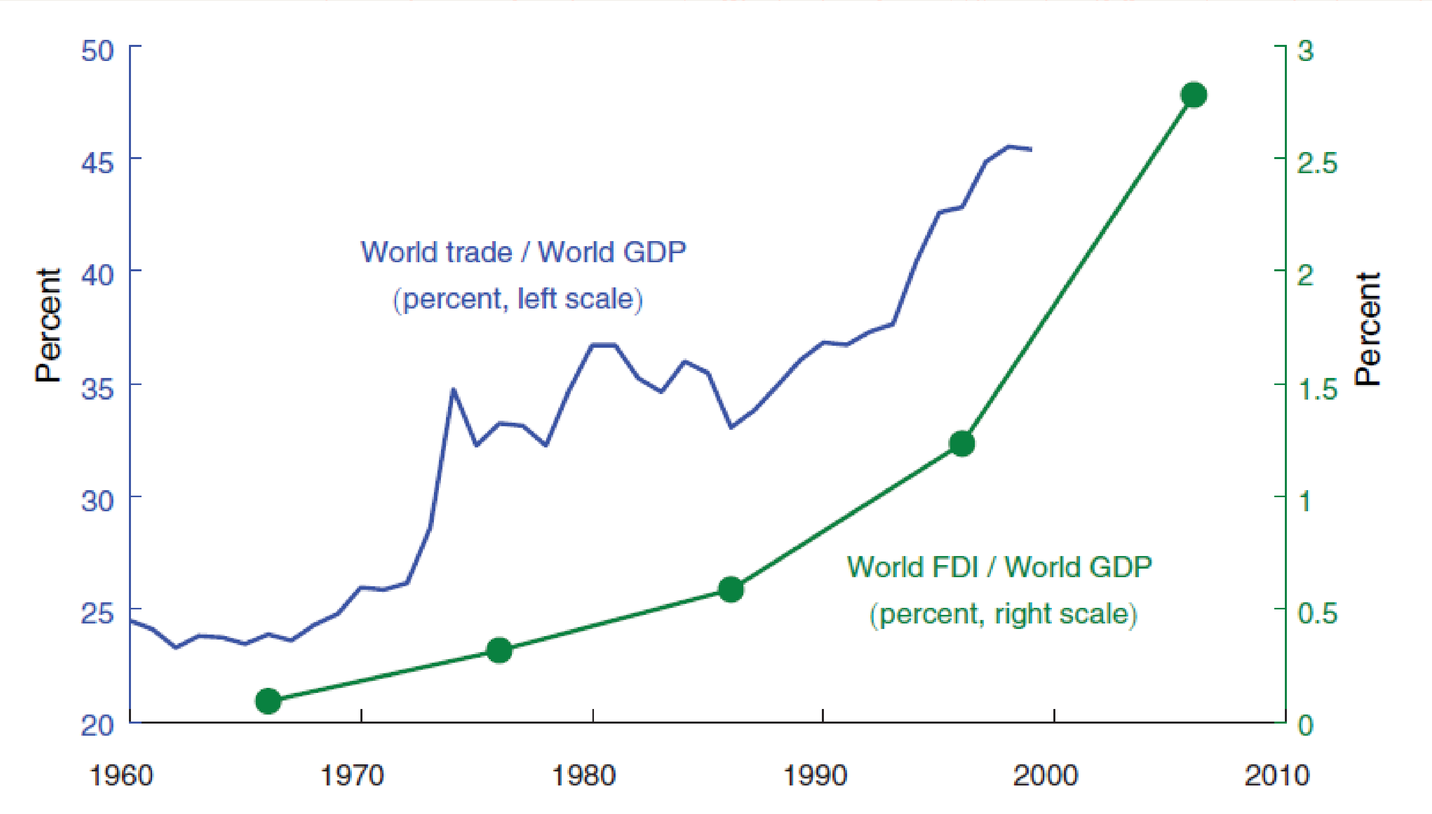
Foundations of Modern Growth Theory

- One of the great accomplishments of neoclassical growth theory is that it produced a single model that captured the first five of Kaldor's facts.

- The other great accomplishment was its explicit microeconomic foundations.

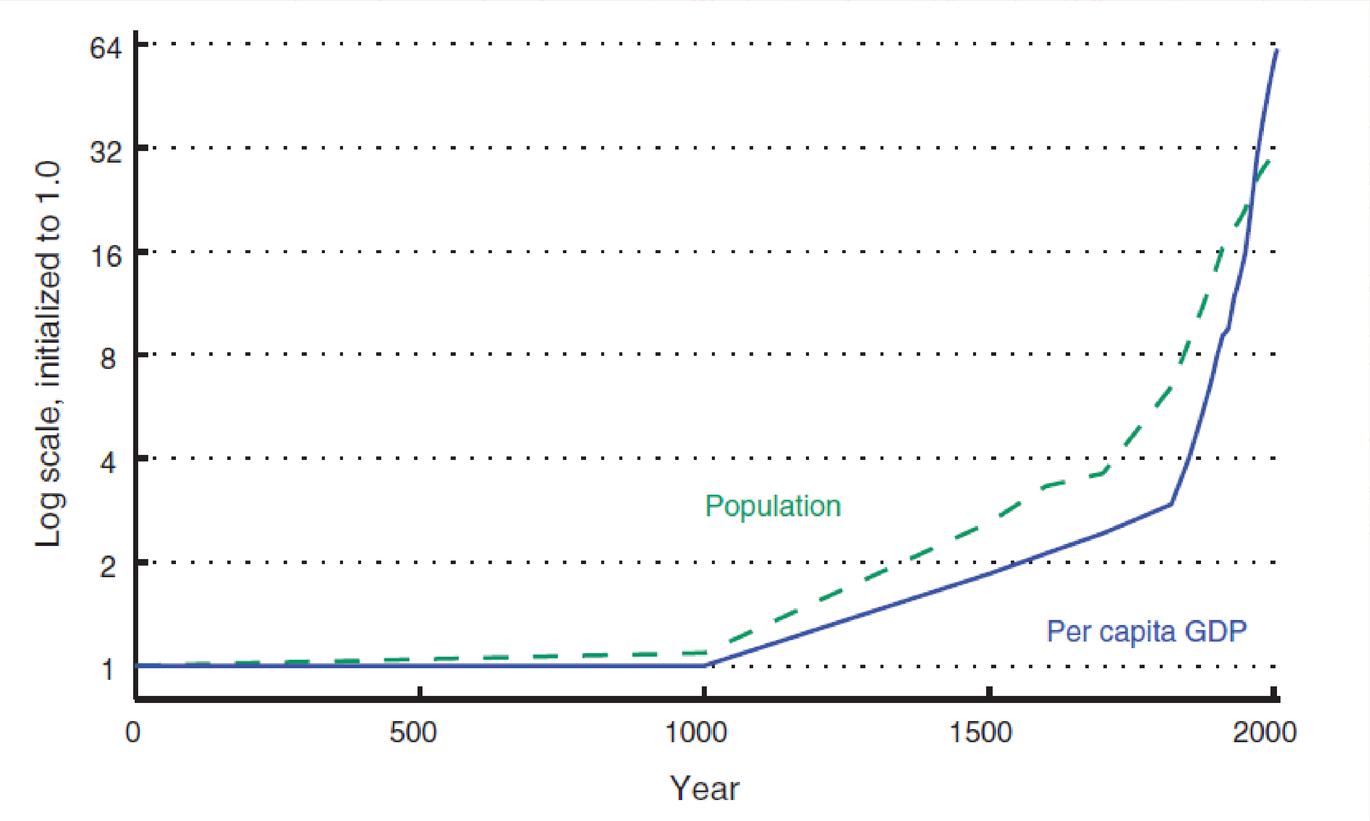
- Jones and Romer propose a new list of stylized facts that are driving the research agenda on economic growth "The New Kaldor Facts"

The Rise in Globalization



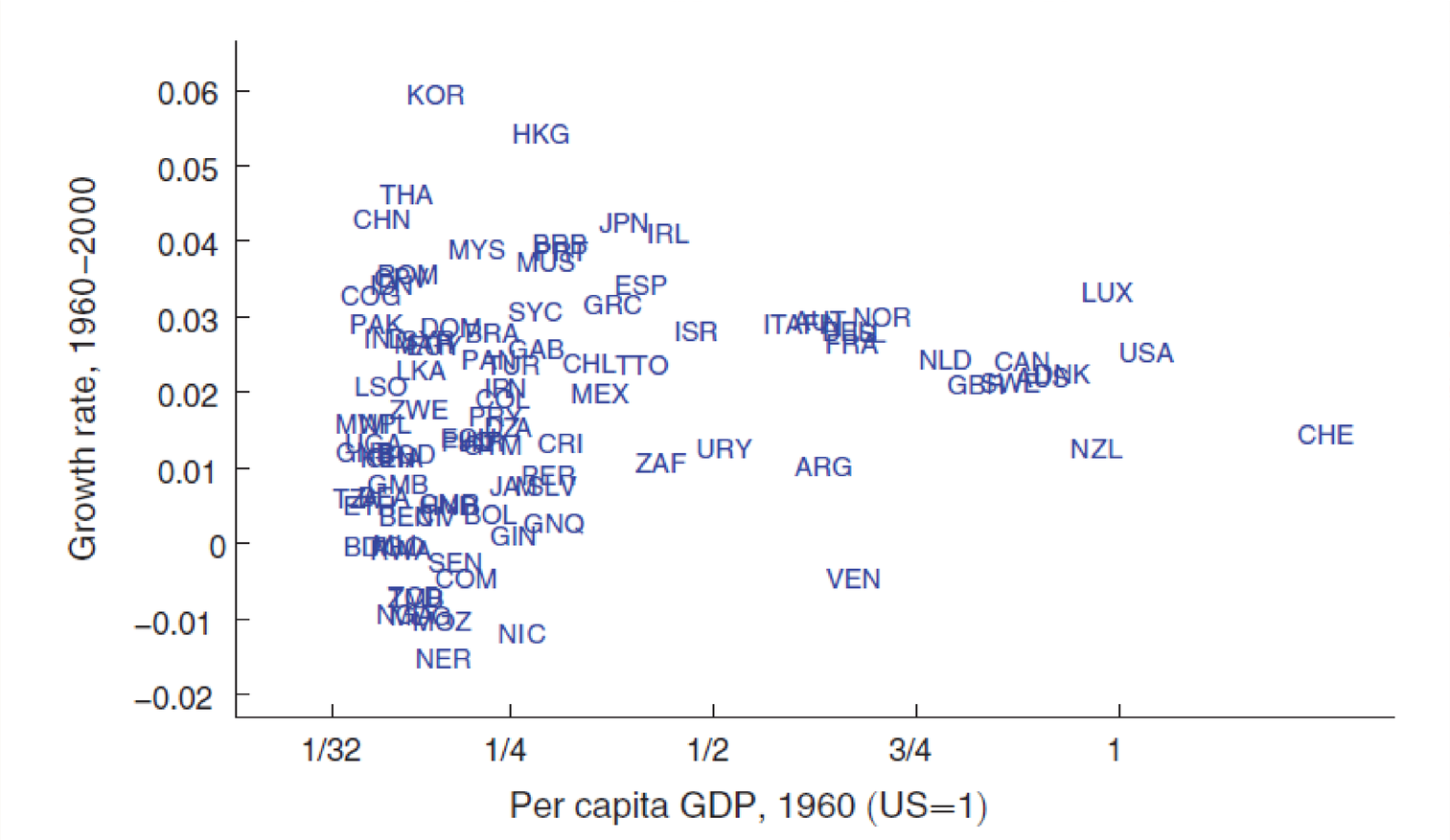
1. **Increases in the extent of the market.**
   - Increased flow of goods, ideas, finance, and people – via globalization, as well as urbanization – have increased the extent of the market for all workers and consumers.

Population and Per Capita GDP over the Very Long Run



2. **Accelerating growth.**
   - For thousands of years, growth in both population and per capita GDP has accelerated, rising from virtually zero to the relatively rapid rates observed in the last century.

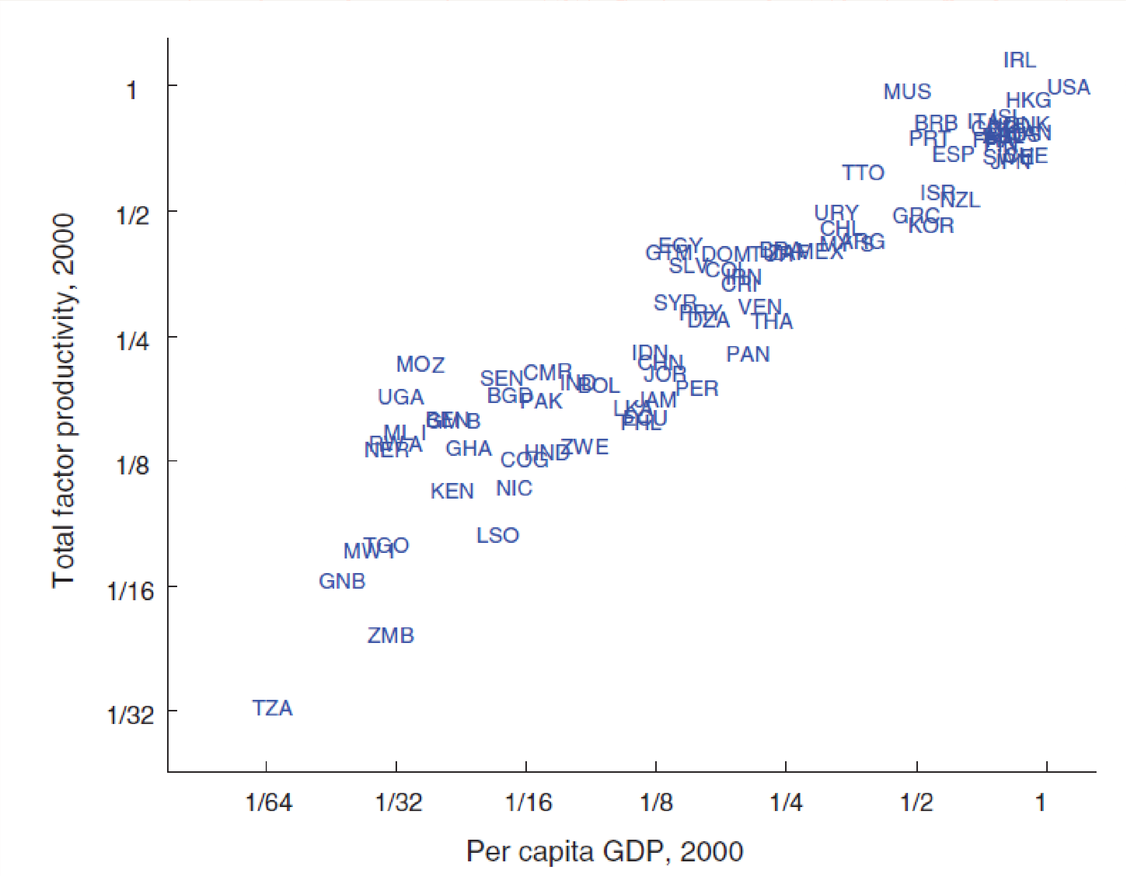
Growth Variation and Distance from the Frontier



3. Variation in modern growth rates.

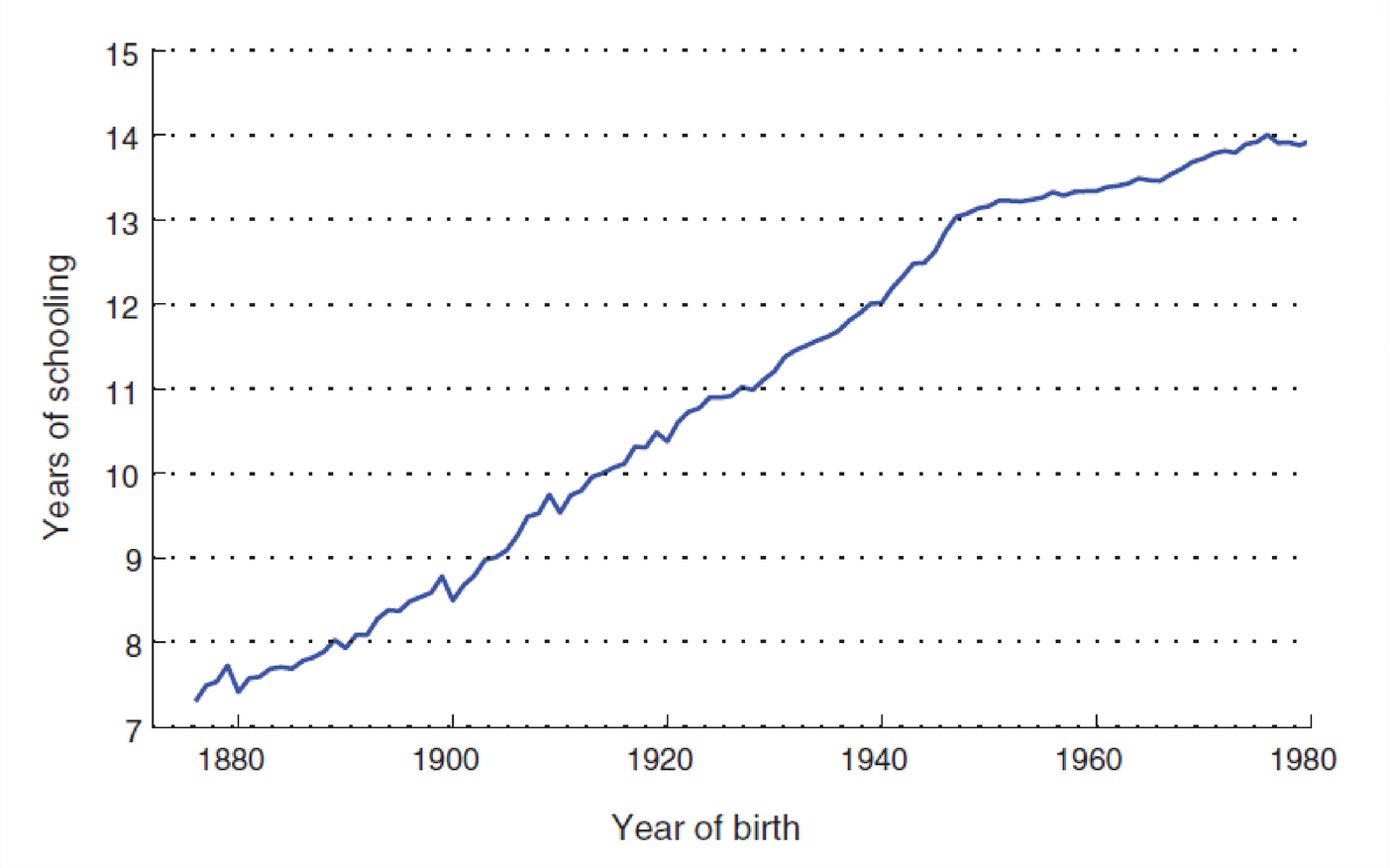
- The variation in the rate of growth of per capita GDP increases with the distance from the technology frontier.

Income and Total Factor Productivity



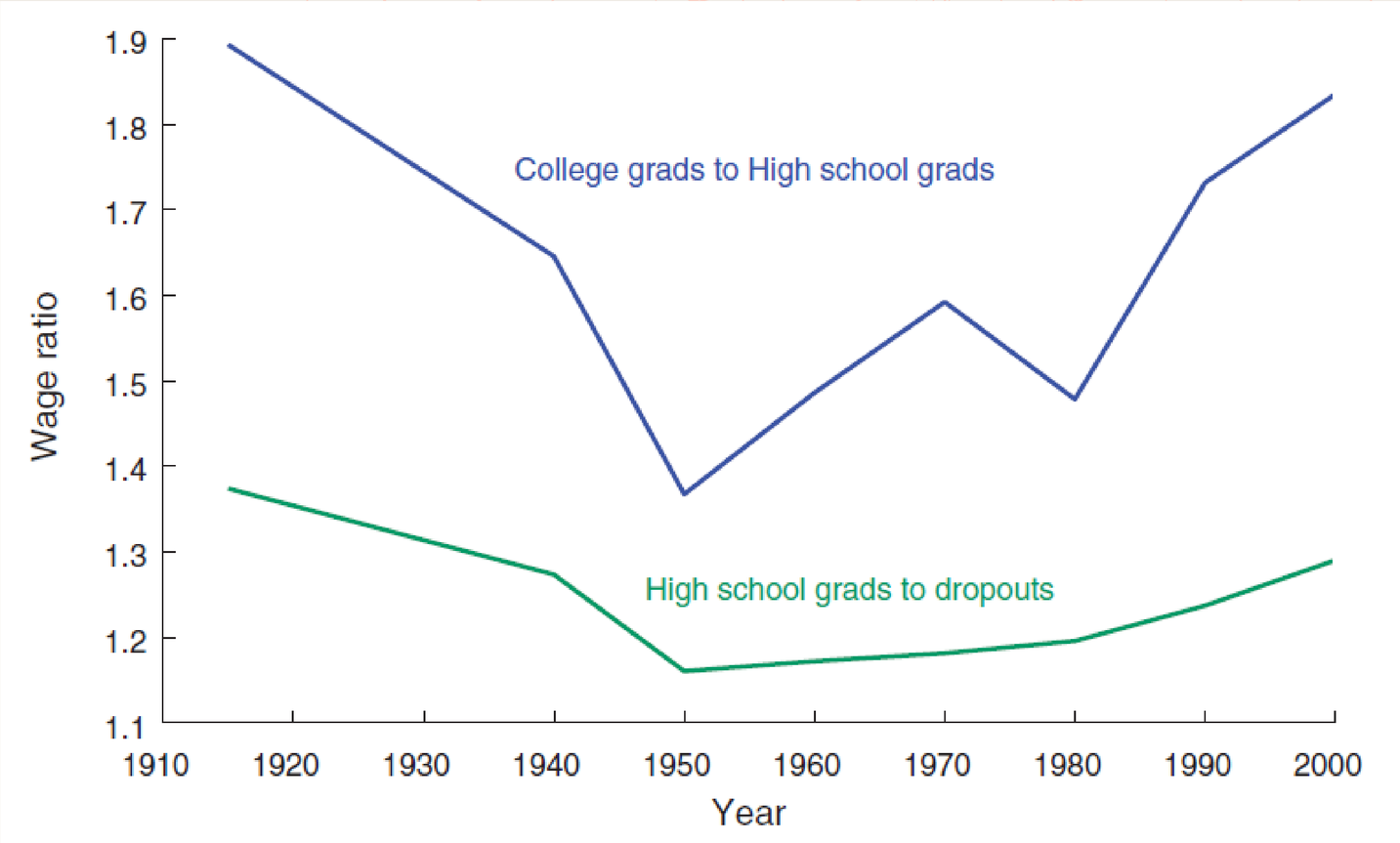
4. **Large income and total factor productivity (TFP) differences.**
   - Differences in measured inputs explain less than half of the enormous cross-country differences in per capita GDP.

Years of Schooling by Birth Cohort in the U.S.



5. **Increases in human capital per worker.**
   - Human capital per worker is rising dramatically throughout the world.

The U.S. College and High School Wage Premiums



6. **Long-run stability of relative wages.**
   - The rising quantity of human capital, relative to unskilled labor, has not been matched by a sustained decline in its relative price.

Jones and Romer's Secondary Goal

➢ Sketch the elements of a theory that could replicate these facts.

- In Solow one endogenous state variable: capital.
- Subsequent contributions have tried to endogeneize the other four state variables: ideas, institutions, population, and human capital.
- To get a model that focuses on the endogenous accumulation of, and interaction between these variables, Jones and Romer suggest to try to endogeneize (¾): ideas, population, and human capital. Meanwhile, keep institutions exogenous in a similar fashion as Solow treated technology.

Ideas and Human Capital

- An idea is a pure nonrival good which can be used by any number of people simultaneously, without congestion or depletion.

- Implications of this distinctive characteristic of ideas:
  - Introduces scale effects.
  - Changes the feasible and optimal economic institutions.

- Human capital is used to produce ideas and ideas are needed to enhance human capital, but human capital is a rival good.

- Increasing returns to scale that are implied by nonrivalry lead to the failure of Adam Smith's invisible hand result.

Standard approach to ideas

Our technology uses ideas A, rival physical goods X, and labor L to produce output Y according to

$$Y = F(A, X, L)$$

constant replication: constant returns to scale to X and L, such that

$$\frac{Y}{L} = F\left(A, \frac{X}{L}, 1\right)$$

with a fixed set of ideas A, average welfare is decreasing in L.

Static benefits from ideas

Now suppose that the stock of ideas is proportional to L. Then

$$\frac{Y}{L} = F\left(L, \frac{X}{L}, 1\right)$$

now two effects of L, a benefit in A, and a cost on X.

$$\frac{Y}{L} = y = mL^{\gamma}$$

if $\gamma > 0$ the benefits outweight the costs and output per worker increases with population. This shows the need to endogeneize ideas to explain fact 1: increase in the extent of the market.

Dynamic benefits from ideas

Now lets analyze the evolution of ideas over time.

Start using a technology given by

$$Y_t = A_t X^{\beta} L_t^{1-\beta} \qquad (2)$$

while we will assume that ideas evolve according to

$$\frac{dA_t}{dt} = \alpha L_t \qquad (3)$$

Case 1: L_t adjust with changes in Y_t such that

$$\frac{Y_t}{L_t} = \bar{y} = 1 \qquad (4)$$

Combining (2) and (4) we get that in equilibrium:

$$\frac{dA_t}{dt} = \alpha A_t^{\frac{1}{\beta}} \qquad (5)$$

Dynamic benefits from ideas

Then

$$\frac{\partial \ln\left(\frac{dA_t}{dt}\right)}{\partial A_t} = \frac{1}{\beta} > 1$$

Useful to explain fact 2: accelerating growth, but unrealistic given that it would require population to keep up with output.

This shows the need to analyze the dynamics of ideas and the need to endogeneize labor.

The Facts of Economic Growth

Charles I. Jones

Charles Jones (2016), "The Facts of Economic Growth," Handbook of Macroeconomics, Volume 2A, Elsevier B. V., Chapter 1: 3-69.

Two Basic Questions

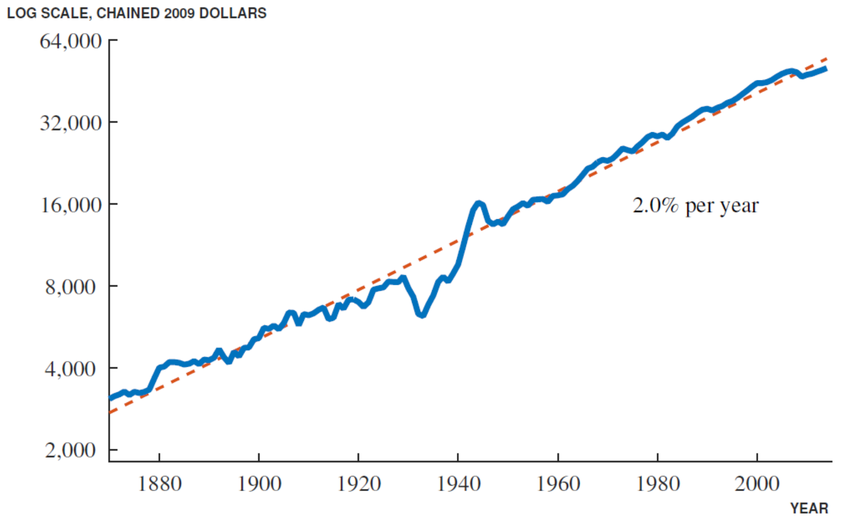
I. How much richer are we today than 100(0) years ago?

II. How large are the income gaps between countries?

- Economics seeks to answer these questions by building quantitative models –models that can be compared with empirical data.
- Growth economics has only partially achieved these goals, but a critical input into our analysis is knowing where the goalposts lie –that is, knowing the facts of economic growth.
- The **goal** of this paper is to lay out as many of these facts as possible. Rather than highlighting a handful of stylized facts, we draw on the last thirty years of the renaissance of growth economics to lay out what is known empirically about the subject.
- The result, I hope, is a fascinating tour of the growth literature from the perspective of the basic data.

I. Growth at the Frontier*
1.1 Modern Economic Growth

Figure 1: GDP per person in the United States



Note: Data for 1929–2014 are from the U.S. Bureau of Economic Analysis, NIPA Table 7.1. Data before 1929 are spliced from Maddison (2008).

Table 1: The Stability of U.S. Growth

Period	Growth Rate	Period	Growth Rate
1870–2007	2.03	1973–1995	1.82
1870–1929	1.76	1995–2007	2.13
1929–2007	2.23		
1900–1950	2.06	1995–2001	2.55
1950–2007	2.16	2001–2007	1.72
1950–1973	2.50		
1973–2007	1.93		

Note: Annualized growth rates for the data shown in Figure 1.

"The Forest"

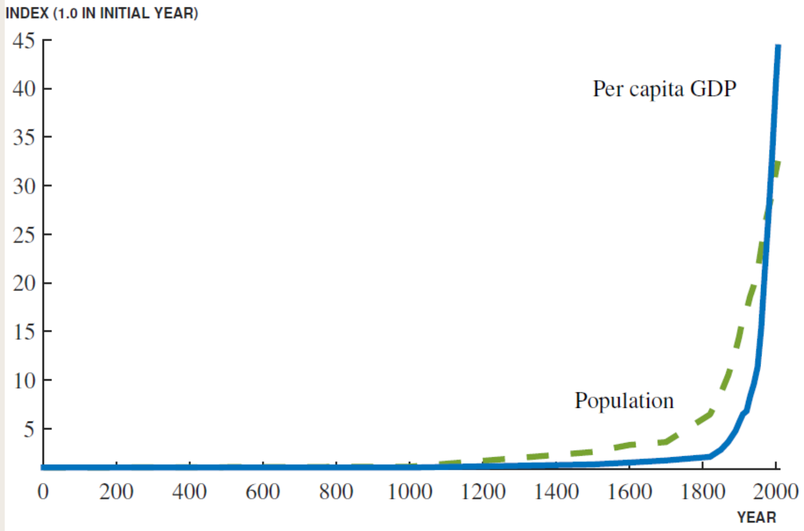
"Trees"

- Steady, sustained exponential growth for the last 150 years or so is a key characteristic of the frontier. All modern theories of economic growth –Solow (1956), Lucas (1988), Romer (1990), Aghion and Howitt (1992)- are designed with this fact in mind.

* By "Frontier" I mean growth among the richest set of countries in any given period.

I. Growth at the Frontier
1.2 Growth over the Very Long Run

Figure 2: Economic Growth over the Very Long Run



Note: Data are from Maddison (2008) for the "West," i.e. Western Europe plus the United States. A similar pattern holds using the "world" numbers from Maddison.

Table 2: The Acceleration of World Growth

Year	GDP per person	Growth rate	Population (millions)	Growth rate
1	590	...	19	...
1000	420	-0.03	21	0.01
1500	780	0.12	50	0.17
1820	1,240	0.15	125	0.28
1900	3,350	1.24	280	1.01
2006	26,200	1.94	627	0.76

Note: Data are from Maddison (2008) for the "West," i.e. Western Europe plus the United States. Growth rates are average annual growth rates in percent, and GDP per person is measured in real 1990 dollars.

- Sustained exponential growth in living standards is an incredibly recent phenomenon (last two centuries).
- Provided the increasing returns associated with ideas [Lee (1988); Kremer (1993); and Jones (2001)] is sufficiently strong to counter the Malthusian diminishing returns, this mechanism can give rise to dynamics like those shown in Figure 2.

II. Sources of Frontier Growth

- One often wishes to look at the data "through the lens of" some growth model that is much simpler than the world that generates the observed data. Following Solow (1957) and (many) others.

$$Y_t = \underbrace{A_t M_t}_{\text{TFP}} K_t^{\alpha} H_t^{1-\alpha} \qquad (1)$$

A_t denotes the economy's stock of knowledge, and M_t is anything else that influences total factor productivity ("measure of ignorance" or "misallocation").

2.1 Growth Accounting

- If one wishes to credit such growth to total factor productivity, it is helpful to do the accounting like, for example, Klenow and Rodríguez-Clare (1997).

$$\frac{Y_t}{L_t} = \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} \frac{H_t}{L_t} \cdot Z_t \qquad \text{where } Z_t \equiv (A_t M_t)^{\frac{1}{1-\alpha}} \qquad (3)$$

- Growth in output per hour $\frac{Y_t}{L_t}$ comes from growth in the capital-output ratio $\frac{K_t}{Y_t}$, growth in human capital per hour $\frac{H_t}{L_t}$, and growth in labor-augmenting TFP, Z_t.

II. Sources of Frontier Growth
2.1 Growth Accounting

Table 3: Growth Accounting for the United States

Period	Output per hour	Contributions from K/Y	Labor Composition	Labor-Aug. TFP
1948–2013	**2.5**	**0.1**	**0.3**	**2.0**
1948–1973	3.3	-0.2	0.3	3.2
1973–1990	1.6	0.5	0.3	0.8
1990–1995	1.6	0.2	0.7	0.7
1995–2000	3.0	0.3	0.3	2.3
2007–2013	1.7	0.1	0.5	1.1

Note: Average annual growth rates (in percent) for output per hour and its components for the private business sector, following equation (3). Source: Authors calculations using Bureau of Labor Statistics, *Multifactor Productivity Trends*, August 21, 2014.

Great productivity period.

Productivity slowdown

Coinciding with the dot-com boom and the rise in the importance of information technology.

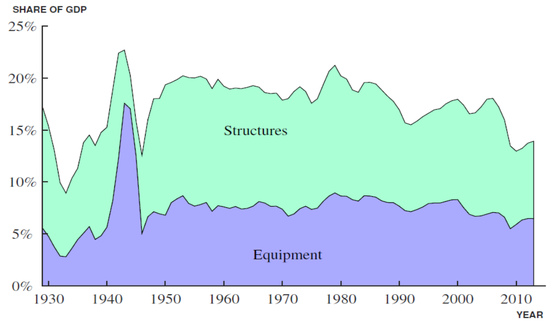
Great Recession?

1. Growth in output per hour at 2.5 percent is slightly faster than the growth in GDP per capita. One reasons is that the BLS data measure growth for the private business sector only.
2. Capital-output ratio is relatively stable over this period, contributing almost nothing to growth.
3. Labor composition (educational attainment, shift from manufacturing to services, and more participation of women in the labor force) contributes in some degree.
4. The "residual" of TFP accounts for the bulk of growth, at 80 percent since 1948.

II. Sources of Frontier Growth
2.2 Physical capital

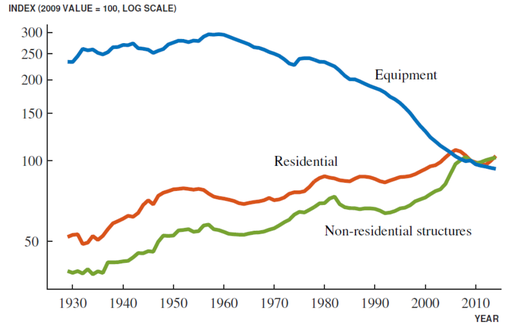
- The fact that the contribution of the capital-output ratio was modest in the growth accounting decomposition suggests that the capital-output ratio is relatively constant over time.
- A fascinating observation is that nominal investment shares are relatively stable when compared to huge trends in relative prices.

Figure 4: Investment in Physical Capital (Private and Public), United States



Source: National Income and Product Accounts, U.S. Bureau of Economic Analysis, Table 5.2.5. Intellectual property products and inventories are excluded. Government and private investment are combined. Structures includes both residential and nonresidential investment. Ratios of nominal investment to GDP are shown.

Figure 5: Relative Price of Investment, United States



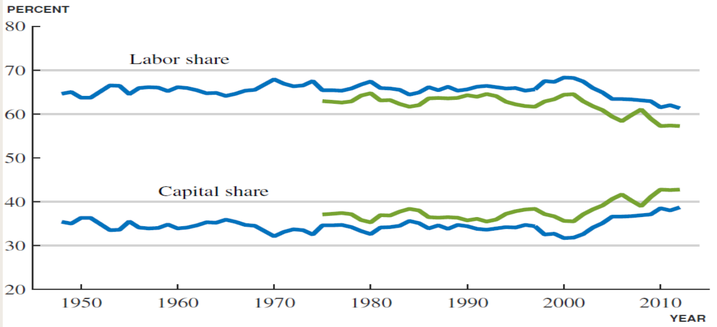
Note: The chained price index for various categories of private investment is divided by the chained price index for GDP. Source: National Income and Product Accounts, U.S. Bureau of Economic Analysis Table 1.1.4.

- Greendwood et al. (1997) and Whelan (2003) point out that one way to reconcile the facts is with a two sector model in which technological progress in the equipment sector is substantially factor that technological progress in the rest of the economy.

II. Sources of Frontier Growth
2.3 Factor shares

- One of the original Kaldor (1961) stylized facts of growth was the stability of the shares of GDP paid to capital and labor.
- Since 2000 or so, there has been a marked decline in the labor share and a corresponding rise in the capital share.

Figure 6: Capital and Labor Shares of Factor Payments, United States

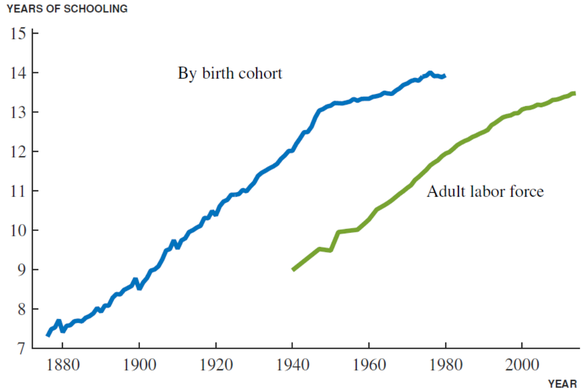


Note: The series starting in 1975 are from Karabarbounis and Neiman (2014) and measure the factor shares for the corporate sector, which the authors argue is helpful in eliminating issues related to self-employment. The series starting in 1948 is from the Bureau of Labor Statistics *Multifactor Productivity Trends*, August 21, 2014, for the private business sector. The factor shares add to 100 percent.

- Is this a temporary phenomenon, perhaps amplified by the Great Recession? Or are some deeper structural factors at work?

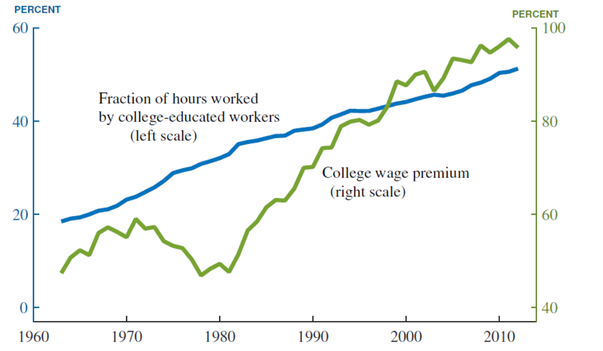
II. Sources of Frontier Growth
2.4 Human capital

Figure 7: Educational Attainment, United States



Note: The blue line shows educational attainment by birth cohort from Goldin and Katz (2007). The green line shows average educational attainment for the labor force aged 25 and over from the Current Population Survey.

Figure 8: The Supply of College Graduates and the College Wage Premium, 1963–2012



Note: The supply of U.S. college graduates, measured by their share of total hours worked, has risen from below 20 percent to more than 50 percent by 2012. The U.S. college wage premium is calculated as the average excess amount earned by college graduates relative to non-graduates, controlling for experience and gender composition within each educational group. Source: Autor (2014), Figure 3.

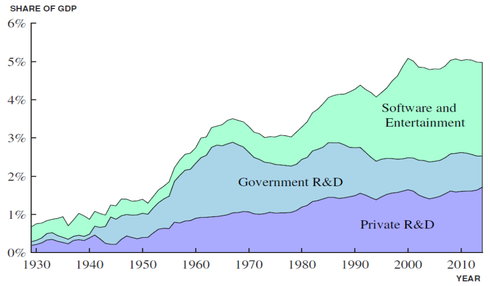
Figure 7 shows two facts. First, for 75 years, educational attainment rose steadily, at a rate slightly less than one year per decade. Second, the leveling-off of educational attainment.

Figure 8 shows stylized facts noted by Katz and Murphy (1992). Even though the supply of college graduates was growing rapidly, the wage premium for college graduates was increasing sharply as well.

II. Sources of Frontier Growth
2.5 Ideas

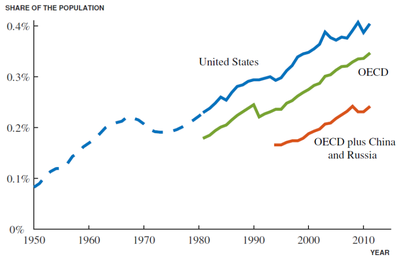
- The production of new ideas (*A* in the production function) plays a fundamental role in the modern understanding of growth [Romer (1990); Grossman and Helpman (1991); and Aghion and Howitt (1992)].

Figure 9: Research and Development Spending, United States



Source: National Income and Product Accounts, U.S. Bureau of Economic Analysis via FRED database. "Software and Entertainment" combines both private and public spending. "Entertainment" includes movies, TV shows, books, and music.

Figure 10: Research Employment Share

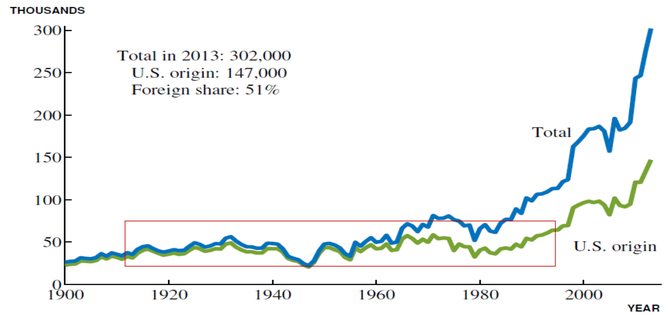


Source: Data for 1981–2001 are from *OECD Main Science and Technology Indicators*, http://stats.oecd.org/Index.aspx?DataSetCode=MSTLPUB. Data prior to 1981 for the United States are spliced from Jones (2002), which uses the NSF's definition of "scientists and engineers engaged in R&D."

- Figure 9 (R&D spending) and Figure 10 (Research employment) show the rapid growth in the input side of the idea production function.

- Figure 11 shows that the number of patents granted to U.S. inventors in 1915, 1950, and 1985 was approximately the same.

Figure 11: Patents Granted by the U.S. Patent and Trademark Office



Source: http://www.uspto.gov/web/offices/ac/ido/oeip/taf/h_counts.htm

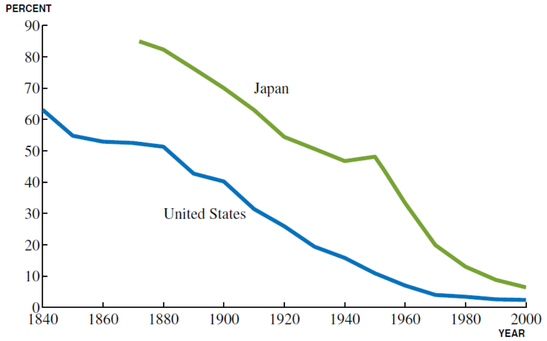
II. Sources of Frontier Growth
2.6 Misallocation

- One of the great insights of the growth literature in the last 15 years is that misallocation at the micro level can show up as a reduction in TFP at a more aggregated level [Banerjee and Duflo (2005); Chari et al. (2007); Restuccia and Rogerson (2008); and Hsieh and Klenow (2009)].
- When resources are allocated optimally, the economy will operate on its production possibilities frontier. When resources are misallocated, the economy will operate inside this frontier.
- Hsieh et al. (2013) show that innate talent for highly-skilled professions is unlikely to differ across groups, the occupational distribution in 1960 suggests that a large number of innately talented African Americans and white women were not working in the occupations dictated by comparative advantage. The paper quantifies the macroeconomic consequences of the remarkable convergence in the occupational distribution between 1960 and 2008 and finds that 15 to 20 percent of growth in aggregate output per worker is explained by the improved allocation of talent.
- Another potential source of misallocation is related to the economics of ideas. To the extent that knowledge spillovers are increasingly internalized or addressed by policy –or to the extent the opposite is true- the changing misallocation of knowledge resources may be impacting economic growth.

III. Frontier Growth: Beyond GDP
3.1 Structural Change

- A type of structural transformation that has seen renewed interest is the possibility that machines (capital) may substitute for labor. Autor et al. (2003) emphasize a polarization, with computerization being particularly substitutable for routine cognitive tasks that can be broken into specific rules (bank tellers) but complementary to nonroutine cognitive tasks (computing programmers) and leaving untouched manual jobs like janitorial work.
- Brynjolfsson and McAfee (2012) highlight broader ramifications of artificial intelligence: computers might start driving cars, reading medical tests, and combing through troves of legal documents.
- What impact will such changes have on the labor market? One useful reference point is the enormous transformation that occurred as the agricultural share of the U.S. labor force went from 2/3 in 1840 to only 2.4 percent by 2000. By and large this transformation was overwhelmingly beneficial.

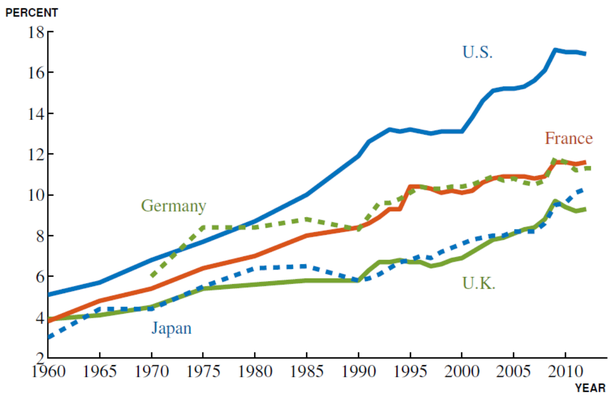
Figure 12: Employment in Agriculture as a Share of Total Employment



Source: Herrendorf, Rogerson and Valentinyi (2014).

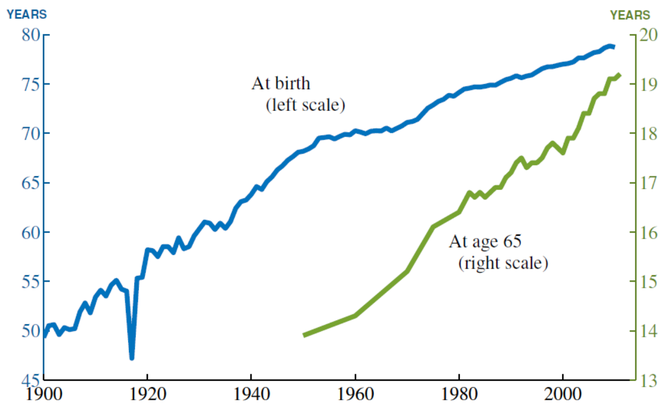
III. Frontier Growth: Beyond GDP
3.2 The Rise of Health

Figure 13: Health Spending as a Share of GDP



Source: *OECD Health Statistics 2014.*

Figure 14: Life Expectancy at Birth and at Age 65, United States

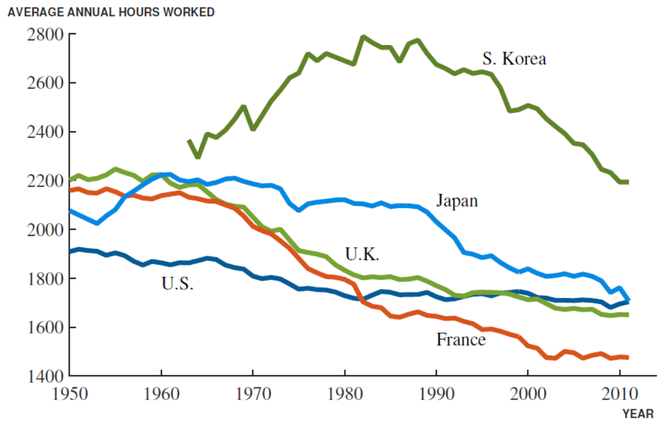


Source: *Health, United States 2013* and https://www.clio-infra.eu.

- Hall and Jones (2007) propose that the widespread rise in the prominence of health care is a byproduct of economic growth. With standard preferences, the marginal utility of consumption declines rapidly. Assuming CRRA preferences, as we get richer and richer, the marginal utility of consumption on any given day declines rapidly; what people really need are more days of life to enjoy their level of consumption.

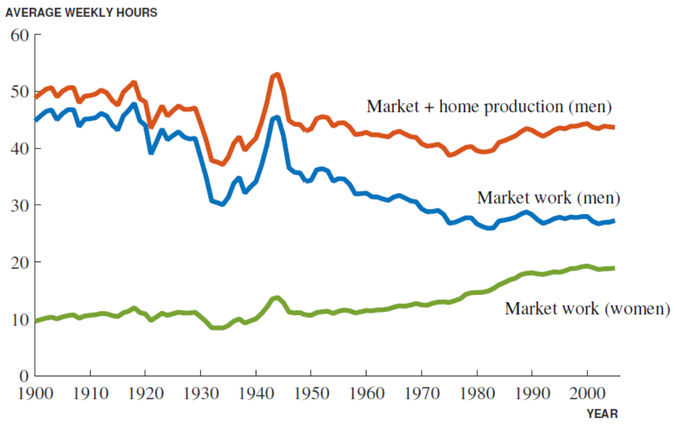
III. Frontier Growth: Beyond GDP
3.3 Hours Worked and Leisure

Figure 15: Average Annual Hours Worked, Select Countries



Source: Average annual hours worked per person employed, from the Penn World Tables 8.0.

Figure 16: Average Weekly Hours Worked, United States



Source: Average weekly hours per worker, from Ramey and Francis (2009).

- Figure 15 (Penn World Tables 8.0) shows that among advanced countries, annual hours worked has fallen significantly since 1950. In the U.S. from 1909 to 1704 annual hours worked in 2011.

- Figure 16 [Ramey and Francis (2009)] breaks the U.S. evidence down into more detail. They show that men are substituting away from market work into home production; which rose from just 4 hours per week in 1900 to more than 16 hours per week in 2005. Thus, the increase in leisure was much smaller than the decline in market hours suggest.

III. Frontier Growth: Beyond GDP

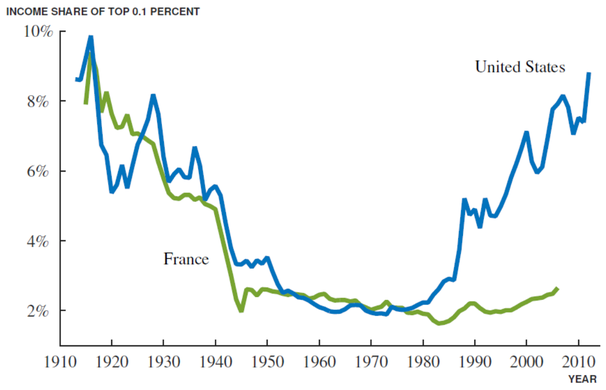
3.4 Fertility

- A large literature seeks to understand the declines in fertility and the hump-shape in population growth that are together known as the demographic transition. A key part of the standard explanation is that children are themselves time intensive, in which case conserving on children also conserves on time as people get richer.

3.5 Top Inequality

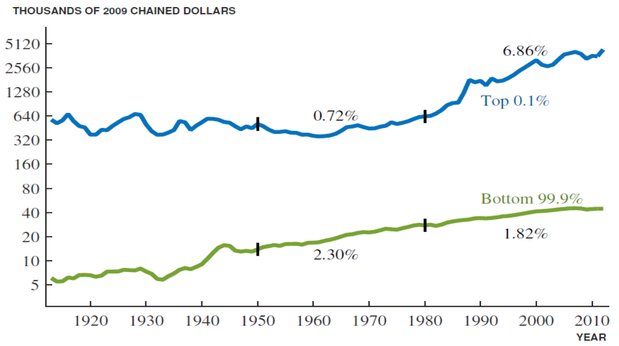
- Piketty and Saez (2003) calculate the income earned by the top 0.1 percent of households. For the U.S., the decline in the first part of the century is associated with capital income, and much of the rise in U.S. inequality since 1980 is associated with labor (and business) income.

Figure 18: Top Income Inequality in the United States and France



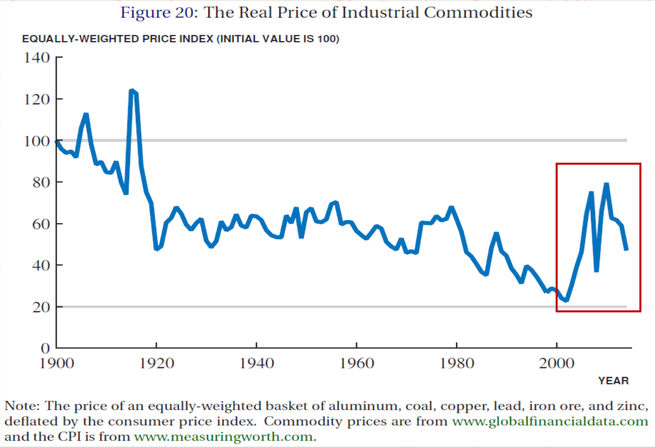
Source: World Top Incomes Database, Alvaredo, Atkinson, Piketty and Saez (2013).

Figure 19: GDP per person, Top 0.1% and Bottom 99.9%



Note: This figure displays an estimate of average GDP per person for the top 0.1% and the bottom 99.9%. Average annual growth rates for the periods 1950–1980 and 1980–2007 are also reported. Source: Aggregate GDP per person data are from Figure 1. The top income share used to divide the GDP is from the October 2013 version of the world top incomes database, from http://g-mond.parisschoolofeconomics.eu/topincomes/.

III. Frontier Growth: Beyond GDP
3.6 The Price of Natural Resources

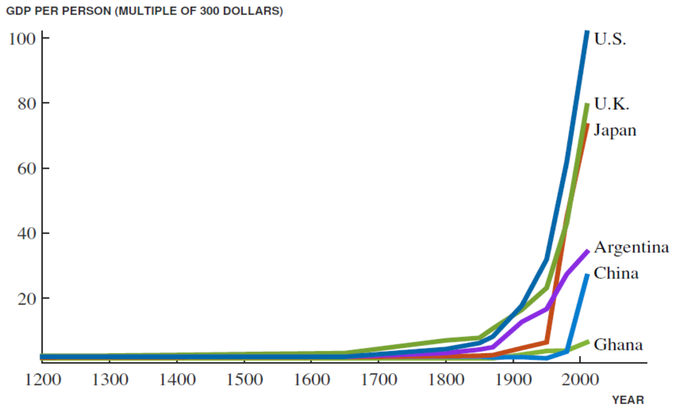


- During the 20th century, world demand for industrial commodities exploded with the rise of the automobile, electrification, urbanization, and the general industrialization that occurred in the U.S. and around the world. The surprise shown in Figure 20 is that the real price of commodities declined over the 20th century.
- Since 2000 we observe a large increase in the real price of these commodities. Many developing countries performed quite well in the 2000s. Some of that growth contributed to the rise in demand for commodities, but some of that success may also reflect commodity-driven growth resulting from the rise in demand from China and India.

IV. The Spread of Economic Growth*
4.1 The Long Run

- The spread of growth over the very long run occurred at different points in time: "The Great Divergence".
- GDP per person in the year 1300 ranges from a high $1,620 in the Netherlands (in 1990 dollars) to $300, given that Pritchett (1997) noticed that the poorest countries in the world in 1950 had that income, and this level seems very close to the minimum average income likely to prevail in any economy at any point in time.
- Therefore the ratio of the richest country to the poorest was 5 in 1300, 10 by 1870 (for the United Kingdom), and more than 100 by 2010 (United States).

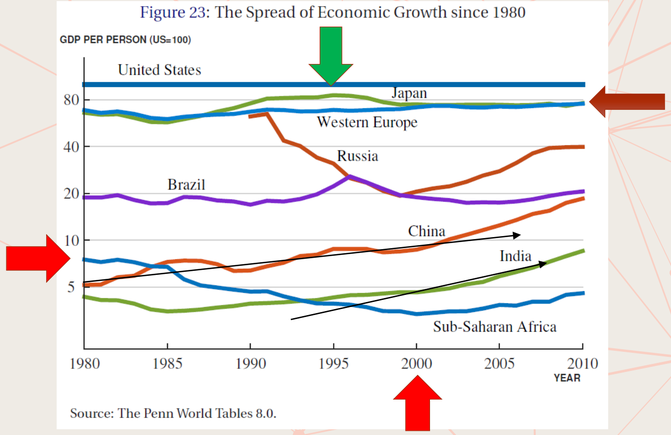
Figure 21: The Great Divergence



Note: The graph shows GDP per person for various countries, normalized by the value in the United Kingdom in the initial year. Source: The Maddison Project, Bolt and van Zanden (2014).

* How are different countries moving relative to the frontier?

IV. The Spread of Economic Growth
4.2 The Spread of Growth in Recent Decades



- Jones and Klenow (2015) note that in addition to the higher leisure [Prescott (2004)], Western Europeans tend to have higher life expectancy and lower consumption inequality. Taking all of these factors into account in constructing a consumption-equivalent welfare measure, the Western European countries look much closer to U.S. levels than the simple GDP per person numbers imply.
- Figure 23 also illustrates the "lost decades" that Japan has experienced. Japan peaked at an income relative to the U.S. of 85 percent in 1995; and since then, has fallen back to around 75 percent of the U.S. level.
- China experiences rapid growth since 1980 and India since around 1990.
- Sub-Saharan Africa falls from 7.5 percent of U.S. income in 1980 to just 3.3 percent by 2000.

IV. The Spread of Economic Growth
4.2 The Spread of Growth in Recent Decades

- Barro (1991), Barro and Sala-i-Martin (1992), and Mankiw et al. (1992) provide a key insight into why the convergence patterns appears in OECD and some middle-income countries but not in the world as a whole.
- Countries around the world are converging –but to their own steady-states, rather than to the frontier. If one conditions on determinants of a country's steady state (such as the investment rates in physical and human capital), then one sees that countries below their steady states grow rapidly and those above their steady states grow slowly (or even decline). The rate at which countries converge to their own steady state ("speed of convergence") seems to be around 2% per year ("Barro's iron law of convergence").

Table 4: The Very Long-Run Distribution

"Bin"	Distribution 1960	Distribution 2010	Distribution Long-Run	Years to "Shuffle"
Less than 5 percent	10	25	26	1470
Between 5 and 10 percent	21	13	11	1360
Between 10 and 20 percent	27	14	8	1040
Between 20 and 40 percent	19	16	8	1120
Between 40 and 80 percent	15	19	28	1440
More than 80 percent	8	13	19	1490

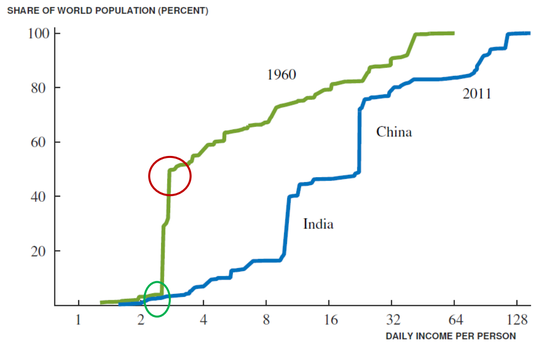
Entries under "Distribution" reflect the percentage of countries with relative (to the U.S.) GDP per person in each bin. "Years to Shuffle" indicates the number of years after which the best guess as to a country's location is given by the long-run distribution, provided that the country begins in a particular bin. Computed following Jones (1997) using the Penn World Tables 8.0 for 100 countries.

- Sorting 100 countries into bins based on their income relative to the frontier and using Markov transition dynamics to calculate sample probabilities that countries move from one bin to another.
- Once countries get on the "growth escalator", good things tend to happen and they grow rapidly towards the frontier. Institutions appear to determine where they end up.
- The very poorest countries seem to be falling further behind.

IV. The Spread of Economic Growth
4.3 The Distribution of Income by Person

- In 1960, 51 percent of the world's population lived on less than 3 dollars per day (measured in 2005 U.S. dollars). By 2011, less than 5 percent of the world's population lived below this level. The big difference, of course, is China and India, which between them contain more than one third of the world's population.

Figure 28: The Distribution of World Income by Population



Source: The Penn World Tables 8.0, calculated across a stable sample of 100 countries.

4.4 Development Accounting

- For development accounting we use the same equation on which we based our growth accounting analysis:

$$\frac{Y_t}{L_t} = \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} \frac{H_t}{L_t} \cdot Z_t \qquad Z_t \equiv (A_t M_t)^{\frac{1}{1-\alpha}} \qquad (6)$$

IV. The Spread of Economic Growth
4.4 Development Accounting

- The Penn World Tables Version 8.0 contains all the information needed to conduct the simplest form of development accounting as in equation 6.
- Key findings:

1. The capital-output ratio is remarkably stable across countries. Caselli and Feyrer (2007) document that the marginal product of capital is very similar in rich and poor countries.
2. The contribution of educational attainment is larger but still modest.
3. Differences in TFP are the largest contributor to income differences in an accounting sense.

- The average country is just over 5 times poorer than the U.S. A factor of around 1.5 (~1.021*1.42) leaving a factor of 3.26 attributed to TFP.

Table 5: Basic Development Accounting, 2010

	GDP per worker, y	Capital/GDP $(K/Y)^{\alpha/(1-\alpha)}$	Human capital, h	TFP	Share due to TFP
U.S.	1.000	1.000	1.000	1.000	...
Hong Kong	0.854	1.086	0.833	0.944	48.9%
Singapore	0.845	1.105	0.764	1.001	45.8%
France	0.790	1.184	0.840	0.795	55.6%
Germany	0.740	1.078	0.918	0.748	57.0%
U.K.	0.733	1.015	0.780	0.925	46.1%
Japan	0.683	1.218	0.903	0.620	63.9%
South Korea	0.598	1.146	0.925	0.564	65.3%
Argentina	0.376	1.109	0.779	0.435	66.5%
Mexico	0.338	0.931	0.760	0.477	59.7%
Botswana	0.236	1.034	0.786	0.291	73.7%
South Africa	0.225	0.877	0.731	0.351	64.6%
Brazil	0.183	1.084	0.676	0.250	74.5%
Thailand	0.154	1.125	0.667	0.206	78.5%
China	0.136	1.137	0.713	0.168	82.9%
Indonesia	0.096	1.014	0.575	0.165	77.9%
India	0.096	0.827	0.533	0.217	67.0%
Kenya	0.037	0.819	0.618	0.073	87.3%
Malawi	0.021	1.107	0.507	0.038	93.6%
Average	0.212	0.979	0.705	0.307	63.8%
1/Average	4.720	1.021	1.418	3.260	69.2%

Computed using the Penn World Tables 8.0 for the year 2010 assuming a common value o $\alpha = 1/3$. The product of the three input columns equals GDP per worker. The penultimate row "Average," shows the geometric average of each column across 128 countries. The "Share due t TFP" column is computed as described in the text. The 69.2% share in the last row is compute looking across the columns, i.e. as approximately 3.5/(3.5+1.5).

IV. The Spread of Economic Growth
4.5 Understanding TFP Differences

- There has been much valuable work on measuring the inputs into development accounting. Lagakos et al. (2012) use household survey data from 35 countries to show that the returns to experience vary substantially across countries; this information boosts the importance of the human capital term by about 50 percent.

4.6 Misallocation: A Theory of TFP

- Suppose output is produced using two tasks (manufacturing and services for the economy as a whole) according to $Y = X_1^{\alpha} X_2^{1-\alpha}$. One unit of labor can produce one unit of either task. Labor could be optimally allocated, or it could be misallocated because of taxes, poor management, information problems, unions, or many other reasons.

- The reduced-form production function is given by: $Y = M(s)L \text{ where } M(s) \equiv s^{\alpha}(1-s)^{1-\alpha}$ (7)
  s is the fraction of labor that works on the first task. The maximizing allocation is $s^* = \alpha$.

- Misallocation provides the theoretical connection between the myriad of distortions in poor economies and the TFP differences that we observe in development accounting.

IV. The Spread of Economic Growth
4.7 Institutions and the Role of Government

- Differences in political and economic institutions have long been conjectured to be fundamental determinants of long-run economic success.
- Olson (1996) observed that history itself provides us with "natural experiments" that allow us to see the large impact of institutions on economic success: North/South Korea after 1953, East and West Germany after WWII, Hong Kong and southeastern China, and across the *Rio Grande* between Mexico and Texas.

Figure 31: Korea at Night

Note: North Korea is the dark area in the center of the figure, between China to the north and South Korea to the south. Pyongyang is the isolated cluster in the center of the picture. Source: http://commons.wikimedia.org/wiki/File:North_and_South_Korea_at_night.jpg

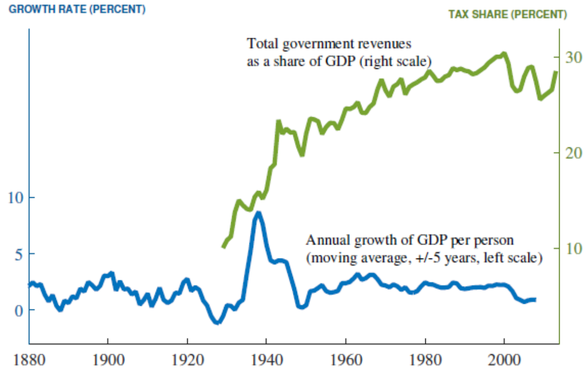
- Acemoglu et al. (2002) and Engerman and Sokoloff (1997) illustrate the "reversal of fortune". In places that were economically successful, Europeans tended to set up "extractive" institutions to transfer the economic gains back to Europe. In contrast, Europeans themselves migrated to places that were sparsely populated, setting up "European" institutions that were conductive to long-run economic success.

IV. The Spread of Economic Growth
4.8 Taxes and Economic Growth

- It is easy to write down models in which governments that tax heavily reduce long-run success of their economies. The facts, however, are not so clear.
- Figure 33 shows that taxes in the U.S. have increased enormously from around 10 percent of GDP in 1929 to more than 30 percent of GDP at their peak in 2000. But, as noted earlier, growth rates over the 20th century were remarkably stable.

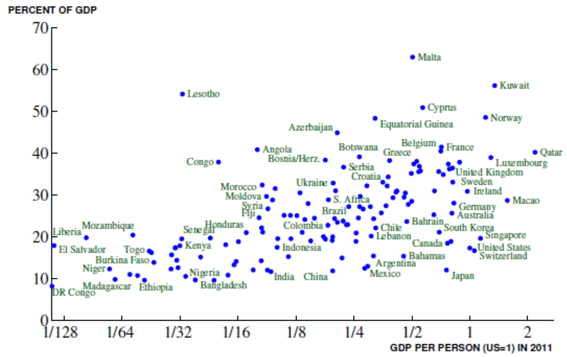
- Figure 34 shows that across countries tax revenues as a share of GDP are positively correlated with economic success. Governments' revenue is used to fund purposes like: providing a stable rule of law, a judicial system, education, public health, highways, basic research, and so on.

Figure 33: Taxes and Growth in the United States



Note: This graph updates a similar figure in Stokey and Rebelo (1995). Total government current receipts are from NIPA Table 3.1 via the FRED database and include federal, state, and local revenues. Real GDP per person is constructed as in Figure 1. The growth rate is smoothed by taking a moving average across the 5 years before and after the relevant date.

Figure 34: Tax Revenues as a Share of GDP



Note: Tax revenue is averaged for the available years between 2000 and 2014, is for the central government only, and includes receipts for social insurance programs. This is an updated graph of a figure from Acemoglu (2005). Source: The World Bank, *World Development Indicators*. GDP per person is from the Penn World Tables 8.0.

IV. The Spread of Economic Growth
4.9 TFPQ versus TFPR

- To measure true productivity, one needs detailed information on micro level prices.

- The economy consists of a unit measure of heterogeneous varieties that enter the utility function via a CES aggregator:

$$C = \int_0^1 (\alpha_i Y_i)^\rho di \qquad (8)$$

Where α_i are taste parameters related to each variety and $0 < \rho < 1$ governs the elasticity of substitution between varieties.

- Each variety is assumed to be produced by different monopolistically-competitive firms using labor:

$$Y_i = A_i L_i \qquad (9)$$

Where A_i is the (exogenous) productivity with which each variety is produced. Assume labor is homogenous and can be hired by any firm at a wage rate ω.

- The price charged by each firm for their variety is a markup over marginal cost:

$$p_i = \frac{1}{\rho} \cdot \frac{w}{A_i}. \qquad (10)$$

- This implies that sales revenue for each firm is $p_i Y_i = \omega \frac{L_i}{\rho}$

IV. The Spread of Economic Growth
4.9 TFPQ versus TFPR

- In the absence of firm-level prices, one typically recovers: $\frac{p_i Y_i}{L_i} = \frac{\omega}{\rho}$; Revenue Productivity.

- The marginal revenue product of capital should also be equated across firms in a simple model like this one, so weighted averages of the average revenue products of capital and labor –which is what Total Factor Productivity Revenue (TFPR) is- should be equated across firms.

- Knowledge of the utility function allows one to compute the marginal rate of substitution between different products.

- The demand curve from utility maximization of (8) is: $$p_i = \rho \alpha_i^{\rho} Y_i^{\rho-1}. \quad (11)$$

- Therefore, sales revenue for this variety is: $$p_i Y_i = \rho(\alpha_i Y_i)^{\rho} \quad (12)$$

- And we can invert this equation to recover the term that enters utility from sales revenue:

$$\alpha_i Y_i = \left(\frac{p_i Y_i}{\rho}\right)^{1/\rho}. \quad (13)$$

True Productivity, TFPQ$_i$: $\frac{\alpha_i Y_i}{L_i} = \frac{(p_i Y_i)^{1/\rho}}{L_i} = \alpha_i A_i.$

- This is the Quantity Total Factor Productivity (TFPQ) that we require to know how effective a firm is at taking a unit of labor (in this case with no capital) and using it to produce different products (valid comparison).

IV. The Spread of Economic Growth
4.10 The Hsieh-Klenow Facts

- Hsieh and Klenow (2009) consider a plant that produces with a Cobb-Douglas production function, using capital and labor, and that faces distortions (taxes, credit market frictions, hiring and firing costs, quantity restrictions, and so on) τ_K and τ_L in choosing its inputs.

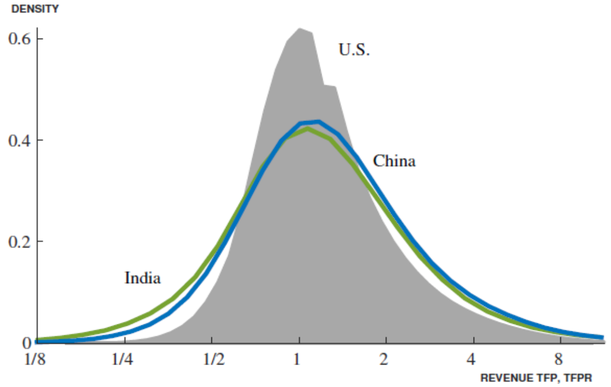
- The profit-maximizing firm will hire capital and labor until the marginal revenue product of these factors equals their net-of-distortion rental price.

- To recover τ_K and τ_L the authors assume that all firms (4-digit industry) have common α_K and α_L Cobb-Douglas exponents. Then variation in factor shares across plants reflects distortions rather than technologies.

- TFPR is not equal to one for every firm, not even in the U.S.: (i) Resources are misallocated even in the U.S., and/or (ii) measurement error in the U.S. data.

- The dispersion of TFPR in India and China is significantly larger. If they had the same dispersion as the U.S., their aggregate TFP would be higher by 30-50% in China and 40-60% in India.

Figure 36: The Distribution of TFPR in 4-digit Manufacturing Industries

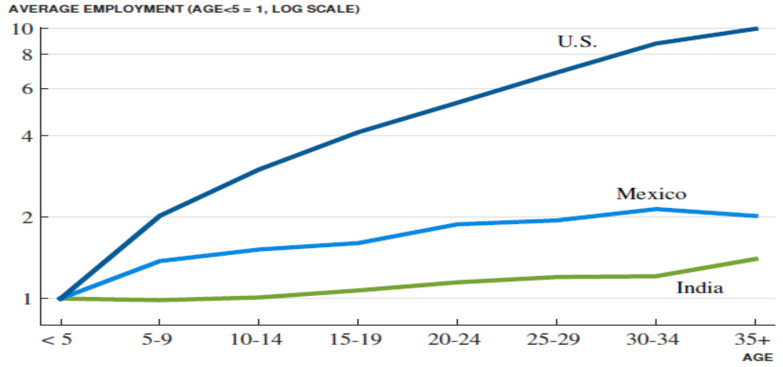


Note: This is the average distribution of TFPR within 4-digit manufacturing industries for the U.S. in 1997, China in 2005, and India in 1994, computed as described in the text. Source: Hsieh and Klenow (2009); data provided by Chang Hsieh.

IV. The Spread of Economic Growth
4.10 The Hsieh-Klenow Facts

- Hsieh and Klenow (2014) look at how establishments in the United States, India, and Mexico grow as they age.
- They conclude that plants that are more than 35 years old in the U.S. have more than 8 times the employment of plants that are less than 5 years old.
- In contrast, old plants in Mexico are only twice as large as young plants, while plants in India exhibit even less employment growth.
- The suggestion is that distortions in Mexico and India prevent the most productive plants from growing in size, and this is one cause of the lower aggregate TFP in these economies. They estimate that moving from the U.S. life cycle to the Indian or Mexican life cycle of plant growth could reduce aggregate TFP by about 25 percent.

Figure 37: Average Employment over the Life Cycle



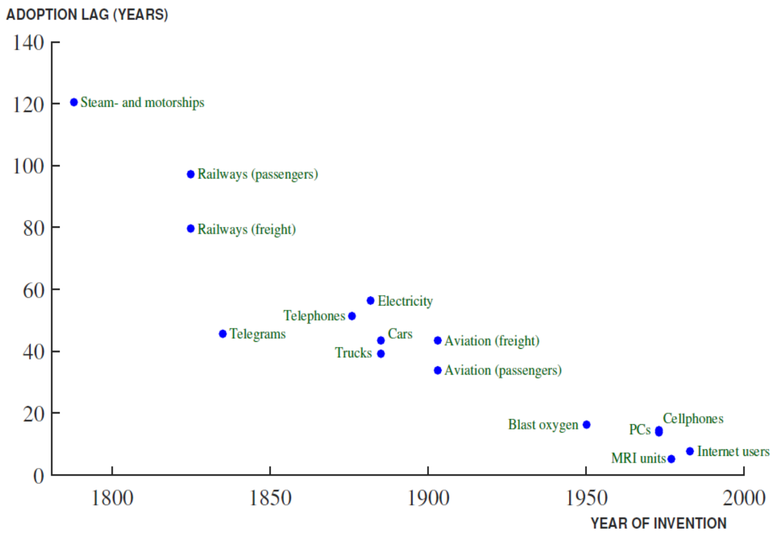
Note: The graph compares average employment per surviving plant in a later year to average employment per operating plant in an earlier year from the same cohort using census data for the manufacturing industry in the United States, Mexico, and India. Source: Hsieh and Klenow (2014); data provided by Chang Hsieh.

IV. The Spread of Economic Growth

4.11 The Diffusion of Ideas

Comin and Hobjin (2010) estimate that technologies invented ten years later are on average adopted 4.3 years faster.

Figure 38: Technology Adoption is Speeding Up Over Time

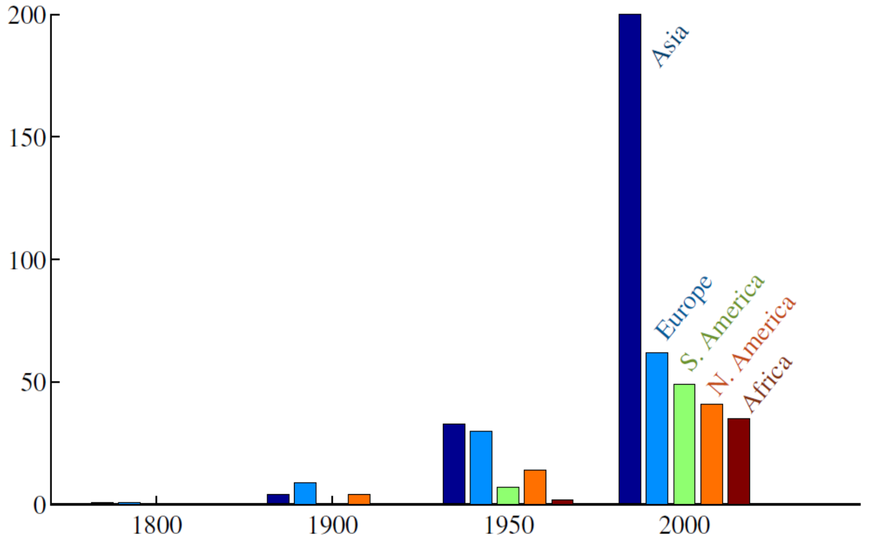


Note: Adoption lags for each country measure the amount of time between when a technology is invented and when it was adopted in the country. The figure reports averages estimated across 166 countries spanning the period 1820 to 2003. Source: Comin and Hobijn (2010)

4.12 Urbanization

Urbanization is even stronger in Asia

Figure 39: The Number of "Million Cities"



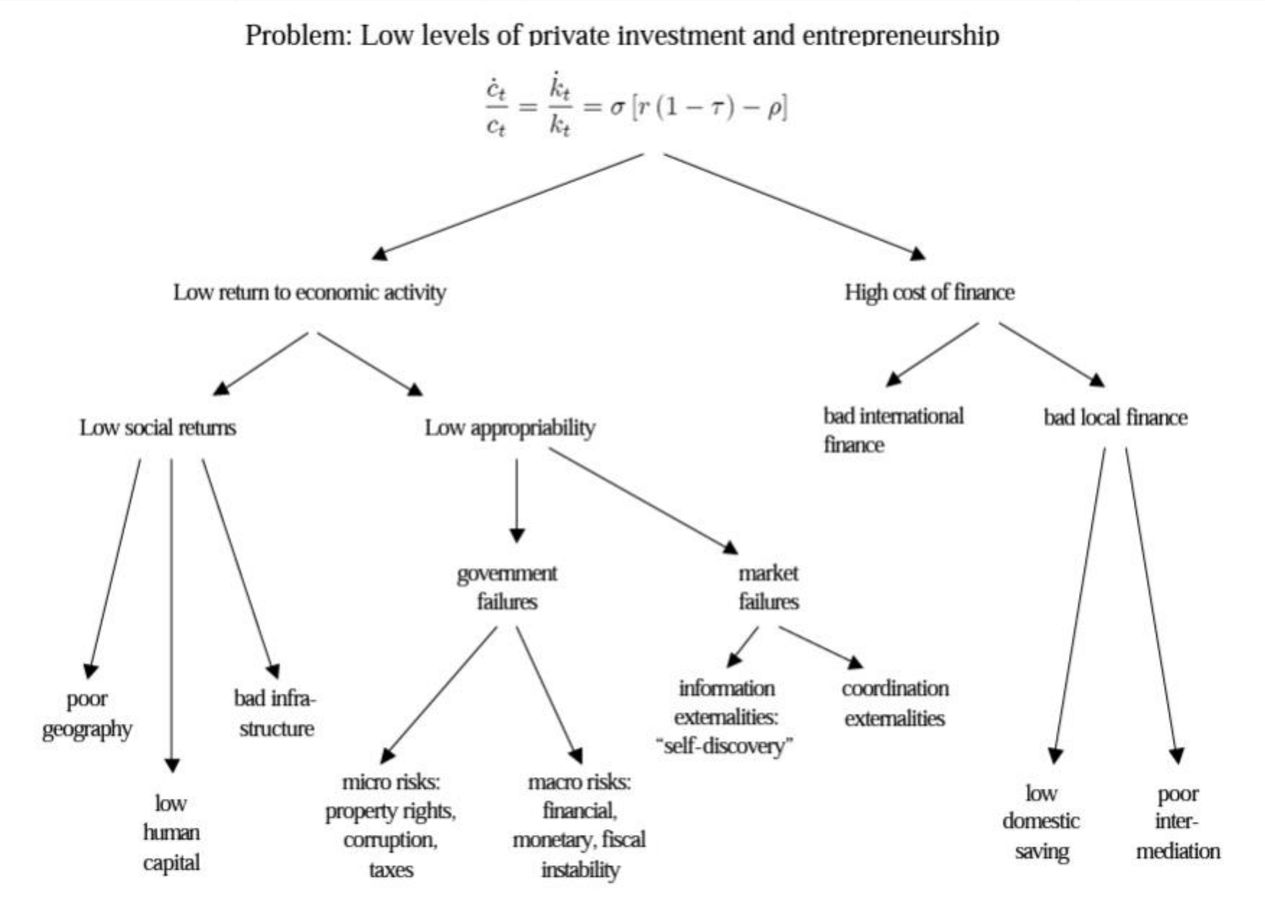
Note: The histogram shows the number of cities on each continent with populations greater than 1 million. Oceania is included with Asia. Source: Satterthwaite (2005), Table 3.

V. Conclusion

- One of the important facts omitted here is globalization and its effects on growth: the decline of trade barriers, the decline in transportation and communication costs, the rise of vertical supply chains.

- It is relatively clear that “institutions matter”. But how important are different institutions, and how do institutions change? Is democracy conductive to growth, or does democracy typically result from growth, or both (Barro, 1999; Acemoglu et al., 2014)? Does human capital accumulation lead to good institutions, or vice verse, or both (Glaeser et al., 2004)? What is the relationship between culture, “fractionalization”, institutions, and growth (Alesina et al., 2003)?

- Another fact we would like to know more about is the extent of knowledge spillovers across countries. Eaton and Kortum (1999) represent one attempt and suggest that only 60% of U.S. growth in recent decades comes from knowledge created in the United States, and the numbers for local knowledge in Japan (35%) and the U.K. (13%) are even smaller.

Growth Diagnostics

- Ricardo Hausmann, Dani Rodrik and Andrés Velasco (2005), "Growth Diagnostics," CID, Harvard University.
- Growth Diagnostics is a methodology to determine the obstacles to a country's capacity to grow. It is a unified framework for identifying the binding constraints to growth, which is key to formulating growth strategies.